Three Phase Controlled Rectifier

- Prof. Dr. Fahmy El-khouly
- Operate from 3 phase ac supply voltage.
- Higher dc output power.
- Higher output voltage ripple frequency.
- Filtering requirements are simplified for smoothing load voltage and load current.
- Extensively used in high power variable speed industrial dc drives.

\[
\begin{align*}
V_{an} &= V_m \sin(\omega t) \\
V_{bn} &= V_m \sin(\omega t - \frac{2\pi}{3}) \\
V_{cn} &= V_m \sin(\omega t + \frac{2\pi}{3})
\end{align*}
\]

\[
\begin{align*}
V_{ab} &= V_{an} - V_{bn} = \sqrt{3} V_m \sin(\omega t + \frac{\pi}{6}) \\
V_{ac} &= V_{an} - V_{cn} = \sqrt{3} V_m \sin(\omega t - \frac{\pi}{6})
\end{align*}
\]
The following are seen from the figure:

From 30° to 150°, $V_{an}$ is maximum.

From 150° to 270°, $V_{bn}$ is maximum.

From 270° to 390°, $V_{cn}$ is maximum.

The following are seen from the figure:

From 30° to 90°, $V_{ab}$ is maximum

From 90° to 150°, $V_{ac}$ is maximum

From 150° to 210°, $V_{bc}$ is maximum

From 210° to 270°, $V_{ba}$ is maximum

From 270° to 330°, $V_{ca}$ is maximum.

From 330° to 390°, $V_{cb}$ is maximum.
Three-Phase Half-Wave Converters

Three single phase half-wave converters can be connected together to form a three phase half-wave converter.

- 3 Phase semiconverters are used in Industrial dc drive applications up to 15 kW power output.
- Known as 3-pulse converter.
- Single quadrant operation is possible.
- The frequency of output voltage is $3f_s$. 
T₁ is triggered at $\omega t = \left(\frac{\pi}{6} + \alpha\right) = (30^0 + \alpha)$

T₂ is triggered at $\omega t = \left(\frac{5\pi}{6} + \alpha\right) = (150^0 + \alpha)$

T₃ is triggered at $\omega t = \left(\frac{7\pi}{6} + \alpha\right) = (270^0 + \alpha)$

Each thyristor conducts for $120^0$ or $\frac{2\pi}{3}$ radians
For Resistive Load

There are two modes of conduction: (i) Continuous conduction mode for $\alpha \leq \pi/6 \ (30^\circ)$, and (ii) Discontinuous conduction mode for $\alpha \geq \pi/6 \ (30^\circ)$

Average output voltage for continuous mode

$$V_{dc} = \frac{1}{2\pi/3} \int \left[ \frac{(5\pi/6) + \alpha}{\pi/6} \right] \cdot V_{an}(\omega t) \, d(\omega t) = \frac{3\sqrt{3}}{2\pi} V_m \cos \alpha$$

The maximum average output voltage that occurs at delay (or firing) angle $\alpha = 0$ is

$$V_{dm} = \frac{3\sqrt{3}}{2\pi} V_m$$

The normalized average output voltage is

$$V_n = \frac{V_{dc}}{V_{dm}} = \cos \alpha$$
RMS output for continuous mode

\[ V_{rms} = \sqrt{\frac{3}{2\pi} \int (\frac{5\pi}{6}+\alpha) \sqrt{\frac{2}{m}} \sin^2 \omega t \, d(\omega t)} = \sqrt{3} V_m \sqrt{\frac{1}{6} + \frac{\sqrt{3}}{8\pi} \cos 2\alpha} \]

Average output voltage for discontinuous conduction \((30^\circ \leq \alpha \leq 150^\circ)\)

\[ V_{dc} = \frac{3}{2\pi} \int (\pi/6 + \alpha) \sqrt{\frac{2}{m}} \sin \omega t \, d(\omega t) = \frac{3V_m}{2\pi} \left[ 1 + \cos(\alpha + \frac{\pi}{6}) \right] \]

The normalized average output voltage is

\[ V_n = \frac{V_{dc}}{V_{dm}} = \frac{1}{\sqrt{3}} \left[ 1 + \cos(\alpha + \frac{\pi}{6}) \right] \]

RMS output voltage for discontinuous conduction \((30^\circ \leq \alpha \leq 150^\circ)\)

\[ V_{rms} = \sqrt{\frac{3}{2\pi} \int (\pi/6 + \alpha) \sqrt{\frac{2}{m}} \sin^2 \omega t \, d(\omega t)} = \sqrt{3} V_m \sqrt{\frac{5}{24} - \frac{\alpha}{8\pi} + \frac{1}{8\pi} \sin(2\alpha + \frac{\pi}{3})} \]
For Highly Inductive Load
Since, the conduction is continuous with inductive load, Average output voltage and RMS output voltage expression are same for the conduction mode with resistive load.

Rectifying Mode: In the case of inductive load the output voltage is continuous and the average value is given positive up to $\alpha = \pi/2$ (90°).

Inverting Mode: If $\alpha > 90^\circ$ then the average value is negative, so that energy is transmitted from the dc circuit to the ac circuit system.
Example 5-8 (10.5): A 3-phase half-wave converter is operated from a 3-phase Y-connected 208 V 60 Hz supply and the load is $R = 10\Omega$. If it is required to obtain an average output voltage of 50% of the maximum possible output voltage. Calculate (i) the delay angle $\alpha$, (ii) the rms and average output currents, (iii) the average and rms thyristor currents, (iv) the rectification efficiency, (v) the transformation utilization factor TUF, and (vi) the input power factor PF.

Solution: The phase voltage is $V_s = 208/\sqrt{3} = 120.1$ V, $V_m = \sqrt{2}V_s = 169.83$ V,

$V_n = V_{dc}/V_m = 0.5$, and $R = 10\Omega$.

The maximum output voltage is: $V_{dm} = \frac{3\sqrt{3}}{2\pi}V_m = \frac{3\sqrt{3} \times 169.83}{2\pi} = 140.45$ V

The average output voltage is: $V_{dc} = V_n V_{dm} = 0.5 \times 140.45 = 70.23$ V

(i) For a resistive load, the load current is continuous if $\alpha \leq (\pi/6)$ and $V_n \geq \cos(\pi/6) = 86.6\%$.

According to given value of $V_n$, the conduction is discontinuous, thus

$0.5 = (1/\sqrt{3})[1 + \cos(\alpha + \pi/6)]$ \hspace{1cm} $\alpha = 67.7^\circ$
(ii) The average output current \( I_{dc} = V_{dc}/R = 7.02 \) V

\[ V_{rms} = \sqrt{3} V_m \sqrt{\frac{5}{24} - \frac{\alpha}{8\pi} + \frac{1}{8\pi} \sin(2\alpha + \frac{\pi}{3})} = 94.74 \text{ V} \]

\[ I_{rms} = V_{rms}/R = 9.47 \text{ A} \]

(iii) The average current of a thyristor, \( I_A = I_{dc}/3 = 2.34 \) A and

The rms current of thyristor, \( I_R = I_{rms}/\sqrt{3} = 5.47 \) A

(iv) Rectification Efficiency, \( \eta = (P_{dc}/V_s I_s) = (70.23 \times 7.02)/(94.74 \times 9.47) = 54.95\% \)

(v) The rms input line current is the same as the thyristor rms current, and the input volt-ampere rating, \( VI = 3V_s I_s \)

\[ TUF = (P_{dc}/V_s I_s) = (P_{dc}/3V_s I_R) = (70.23 \times 7.02)/(3 \times 120.1 \times 5.47) = 25\% \]

(vi) \( PF = (P_o/V_s I_s) = (I_{rms}^2 R/3V_s I_R) = (9.47^2 \times 10)/(3 \times 120.1 \times 5.47) = 0.455 \)
Three-Phase Full Converters

- 3 Phase Fully Controlled Full Wave Bridge Converter.
- Known as a 6-pulse converter.
- Used in industrial applications up to 120kW output power.
- Two quadrant operation is possible.
- The frequency of output voltage is $6f_s$.

The phase shift between the triggering of two adjacent SCRs is $60^\circ$.
EachSCR conducts in two pairs and each pair conducts for $60^\circ$.
When the two SCRs are conducting, i.e. one from positive (upper) group and one from negative (lower) group, the corresponding line voltage is applied across the load.
When the upper SCR conducts, the current of that phase is positive whereas when the lower SCR conducts, the current is negative.
Table 6.2 Firing sequence of SCRs

<table>
<thead>
<tr>
<th>Serial No</th>
<th>$\omega t$</th>
<th>Incoming SCR</th>
<th>Conducting pair</th>
<th>Outgoing SCR</th>
<th>Line voltage across the load</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$30^\circ + \alpha$</td>
<td>$T_1 (a +ve)$</td>
<td>$T_6, T_1$</td>
<td>$T_5$</td>
<td>$v_{ab}$</td>
</tr>
<tr>
<td>2.</td>
<td>$90^\circ + \alpha$</td>
<td>$T_2 (c -ve)$</td>
<td>$T_1, T_2$</td>
<td>$T_6$</td>
<td>$v_{ac}$</td>
</tr>
<tr>
<td>3.</td>
<td>$150^\circ + \alpha$</td>
<td>$T_3 (b +ve)$</td>
<td>$T_2, T_3$</td>
<td>$T_1$</td>
<td>$v_{bc}$</td>
</tr>
<tr>
<td>4.</td>
<td>$210^\circ + \alpha$</td>
<td>$T_4 (a -ve)$</td>
<td>$T_3, T_4$</td>
<td>$T_2$</td>
<td>$v_{ba}$</td>
</tr>
<tr>
<td>5.</td>
<td>$270^\circ + \alpha$</td>
<td>$T_5 (c +ve)$</td>
<td>$T_4, T_5$</td>
<td>$T_3$</td>
<td>$v_{ca}$</td>
</tr>
<tr>
<td>6.</td>
<td>$330^\circ + \alpha$</td>
<td>$T_6 (b -ve)$</td>
<td>$T_5, T_6$</td>
<td>$T_4$</td>
<td>$v_{eb}$</td>
</tr>
</tbody>
</table>

Three Phase Full Converter for Continuous Conduction ($\alpha \leq 60^\circ$)

- At $\omega t = (\pi/6+\alpha)$, thyristor $T_6$ already conducting and $T_1$ is turned-on.
- During interval $(\pi/6+\alpha) \leq \omega t \leq (\pi/2+\alpha)$, $T_1$ and $T_6$ conduct and line-to-line voltage $V_{ab}$ appears across the load.
- At $\omega t = (\pi/2+\alpha)$, $T_2$ is fired and $T_6$ reverse biased immediately.
- During interval $(\pi/2+\alpha) \leq \omega t \leq (5\pi/6+\alpha)$, $T_1$ and $T_2$ conduct and line-to-line voltage $V_{ac}$ appears across the load.
The firing sequence of thyristors is \((T_1, T_2); (T_2, T_3); (T_3, T_4); (T_4, T_5); (T_5, T_6)\) and \((T_6, T_1)\).
As the firing angle $\alpha$ changes from 0 to $90^\circ$, the voltage changes from maximum to zero and the converter is said to be in rectification mode (operate in first quadrant).

The load current is continuous for $0 \leq \alpha \leq \pi/3$ (or $60^\circ$).

The load current is discontinuous for $\alpha > \pi/3$ (or $60^\circ$).
Average output voltage for highly inductive load (continuous conduction for \( \leq \alpha \leq \pi/3 \))

\[
V_{dc} = 6 \times \frac{1}{2\pi} \int_{(\pi/6)+\alpha}^{(\pi/2)+\alpha} \sqrt{3} V_m \sin(\omega t + \pi/6) d(\omega t) = \frac{3\sqrt{3} V_m}{\pi} \cos \alpha
\]

The maximum average output voltage that occurs at delay (or firing) angle \( \alpha = 0 \) is

\[
V_{dm} = \frac{3\sqrt{3}}{\pi} V_m
\]

The normalized average output voltage is

\[
V_n = \frac{V_{dc}}{V_{dm}} = \cos \alpha
\]

RMS output voltage for highly inductive load (continuous conduction for \( \leq \alpha \leq \pi/3 \))

\[
V_{rms} = \sqrt{6 \times \frac{1}{2\pi} \int_{(\pi/6)+\alpha}^{(\pi/2)+\alpha} \left[\sqrt{3} V_m \sin(\omega t + \pi/6)\right]^2 d(\omega t)} = \sqrt{3} V_m \sqrt{\frac{1}{2} + \frac{3\sqrt{3}}{4\pi} \cos 2\alpha}
\]
For the angles in the ranges $90^\circ$ to $180^\circ$, the voltage varies from zero to negative maximum and the converter is said to be in inversion mode (operate in fourth quadrant). For inductive load $\alpha_{\text{max}} = \pi$ (or $180^\circ$).
Waveshapes for Resistive Load
For $\alpha = 2\pi/3$ (or $120^\circ$) with resistive load the output voltage is zero and hence $\alpha_{\text{max}} = 2\pi/3$ (or $120^\circ$).

For $\alpha > \pi/3$ with resistive load, the full converter behaves as semiconverter.

Example 5-10 (10.6): A 3-phase full converter is operated from a 3-phase Y-connected 208 V 60 Hz supply and the load is $R = 10\Omega$. If it is required to obtain an average output voltage of 50% of the maximum possible output voltage. Calculate (i) the delay angle $\alpha$, (ii) the rms and average output currents, (iii) the average and rms thyristor currents, (iv) the rectification efficiency, (v) the transformation utilization factor TUF, and (vi) the input power factor PF.
Solution: The phase voltage is $V_s = \frac{208}{\sqrt{3}} = 120.1$ V, $V_m = \sqrt{2}V_s = 169.83$ V,

$V_n = \frac{V_{dc}}{V_m} = 0.5$, and $R = 10\Omega$.

The maximum output voltage is: $V_{dm} = \frac{3\sqrt{3}}{\pi} V_m = \frac{3\sqrt{3} \times 169.83}{\pi} = 280.90$ V

The average output voltage is: $V_{dc} = V_n V_{dm} = 0.5 \times 280.9 = 140.45$ V

(i) $0.5 = \cos \alpha$; $\alpha = 60^\circ$

(ii) The average output current $I_{dc} = \frac{V_{dc}}{R} = 14.05$ V

$$V_{rms} = \sqrt{3} V_m \sqrt{\frac{1}{2} + \frac{3\sqrt{3}}{4\pi} \cos 2\alpha} = 159.29$$ V

$I_{rms} = \frac{V_{rms}}{R} = 15.93$ A

(iii) The average current of a thyristor, $I_A = \frac{I_{dc}}{3} = 4.68$ A and

The rms current of thyristor, $I_R = \frac{I_{rms}}{\sqrt{(2/6)}} = 9.2$ A
(iv) Rectification Efficiency, $\eta = \left( \frac{P_{dc}}{V_{rms}I_{rms}} \right)$

$\eta = \frac{140.45 \times 14.05}{159.29 \times 15.93} = 77.8\%$

(iv) The rms input line current, $I_s = I_{rms}\left[\sqrt{\frac{4}{6}}\right] = 13\ A$

$TUF = \left( \frac{P_{dc}}{V_s I_s} \right) = \left( \frac{P_{dc}}{3V_s I_s} \right) = \frac{140.45 \times 14.05}{3 \times 120.1 \times 13} = 42.1\%$

(vi) $PF = \left( \frac{P_o}{V_s I_s} \right) = \left( \frac{I_{rms}^2 R}{3V_s I_s} \right) = \frac{15.93^2 \times 10}{3 \times 120.1 \times 13} = 0.542$