Sheet No. (9)

1- Evaluate

\[ I = \int_{0}^{\infty} \int_{y}^{\infty} e^{-x^2} \, dx \, dy \]

2- Evaluate

\[ I = \int_{0}^{1} \int_{x}^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}} \, dy \, dx \]

3- Find the area of the cardioid \( r=a(1+\cos \theta) \)

4- Find the area between \( y=x^2 \) and \( y=x+1 \)

5- Find the volume cut off from the paraboloid \( x^2 + \frac{y^2}{4} + z = 1 \) by the plane \( z=0 \)

6- Evaluate \( \iint \sqrt{1-x^2-y^2} \, dx \, dy \) on the unit circle whose Centre at the origin

7- Evaluate \( \iint \sqrt{1-x^2-a^2-y^2} \, dx \, dy \) on the region \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \)

8- Evaluate the integral \( \int_{0}^{\infty} e^{-x^2} \, dx \)

9- Evaluate the integral \( \iiint_A x^2 \, dx \, dy \, dz \) over the volume of the ellipsoid \( \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \)

10- The geometric model of a material body is a plane region \( R \) bound by \( y=x^2 \) and \( y=\sqrt{2-x^2} \) in the interval \([0,1]\) and with a density function \( r=xy \) (a) Draw the graph of the region. (b) Find the mass of the body. (c) Find the coordinates of the center of mass.