Review (lecture 2) Basic units / Dimensions / Properties

For units: Use SI units (N, kN, mm, m)

For a rectangular R.C. Beam section let:
- \( b \) – width of rectangular R.C. section (Beam sec.)
- \( t \) – total height of R.C. section
- \( d \) – effective depth (distance to c.g of steel)
- \( A_s \) – Area of steel

(Ex.: for 4\( \phi \)18 \( A_s = 4 \times \pi (18)^2 / 4 = 1018 \text{ mm}^2 \))
- \( c \) - cover [50 mm/(1 row)–70/(2 rows)–100/(3 rows)]
- \( E_s \) - Modulus of Elasticity of steel = 200 kN/mm²
- \( E_c \) - Modulus of Elasticity of concrete = 4400\( \sqrt{f_{cu}} \)
- \( n \) – modular ratio \( n = \frac{E_s}{E_c} \) (Code suggests: \( n = 10 \) in deflection & \( M_{cr} \) calculations
- \( n = 15 \) in working stage analysis \( M_{working} \))
- \( f_{cu} \) – characteristic strength of conc. (20 or 25 or 30 N/mm²)
- \( f_y \) - yield stress of steel (240 or 360 or 400 N/mm²)
**Calculation of $M_{cracking}$**

Procedure:
Transform the area of steel to equivalent concrete, nAs using the modular ratio:

$$n = \frac{E_s}{E_c} = 10$$

1- Calculate the location of the N.A. ($e = \overline{\square}$, $y_t = t/2 - e$)

2- Calculate the total moment of Inertia @ N.A.

3- Calculate the cracking moment $M_{cr}$ based on $f_{\text{bottom}} = f_{ctr} = 0.6f_{cu}$

$$M_{cr} = \frac{f_{ctr}I_t}{y_t}$$

---

**Example #1**

Calculate the cracking moment $M_{cr}^{+ve}$ and $M_{cr}^{-ve}$ for the singly reinforced rectangular section shown with dimensions $b \times t = 250 \times 800$ mm, if $A_s = 5 \phi 16$ mm. Materials used are concrete $f_{cu} = 25$ N/mm$^2$ and st. grade 360/520.

**Solution**

a- case of $M_{cr}^{+ve}$

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**Calculation of $M_{cr}^{+ve}$ and $M_{cr}^{-ve}$ for beam sections**

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for $5\phi 16 \quad A_s = 5(\pi \frac{16^2}{4}) = 1005 \ mm^2$

Moments of Areas @ middle line (@ C.L. of concrete section)

$$bt(0) + nA_s \left( \frac{t}{2} - c \right) = (bt + nA_s)(e)$$

$$10(1005)(400 - 50) = [(250)(800) + 10(1005)]e$$

$$e = 16.7 \ mm$$

$$y_t = \frac{t}{2} - e = 400 - 16.7 = 383.3 \ mm$$

$$I = \frac{bt^3}{12} + bt(e)^2 + nA_s(y_t - c)^2$$

$$= \frac{250(800)^3}{12} + 250(800)(16.7)^2 + 10(1005)(383.3 - 50)^2$$

$$= 1.19 \times 10^{10} \ mm^4$$

**Cracking Moment**

$$M_{cr} = \frac{f_{ctr}I}{y_t}$$

$$f_{ctr} = 0.6\sqrt{f_{cu}}$$

$$= 0.6\sqrt{25} = 3 \ N/mm^2$$

$$y_t = 383.3 \ mm$$

$$I = 1.19 \times 10^{10} \ mm^4$$

$$Cracking \ Moment \ M_{cr} = \frac{f_{ctr}I}{y_t} = \frac{3(1.19 \times 10^{10})}{383.3}$$

$$= 9.31 \times 10^7 \ N.mm$$

$$= 93.1 \ kN.m$$
Example #1 - b
- case of \( M_{cr}^{ve} \)

for the rectangular sections of the same dimensions with the same \( A_S \) (in tension side) the positive cracking moment \( M_{cr}^{+ve} = \) the negative cracking moment \( M_{cr}^{-ve} \)

---

Calculation of \( M_{cr}^{+ve} \) and \( M_{cr}^{-ve} \) for beam sections

2- For T-section:

\[
M_{cr} = \frac{f_{ctr} \cdot I_g}{y_t}
\]

a- Negative bending moment

b- Positive bending moment

for the T-sections of the same dimensions with the same \( A_S \) (in tension side) the positive cracking moment \( M_{cr}^{+ve} \neq \) the negative cracking moment \( M_{cr}^{-ve} \)
Example #2

Calculate the cracking moment $M_{cr}^{+ve}$ and $M_{cr}^{-ve}$ for the singly reinforced T-section shown with dimensions $b \times t = 200 \times 700$ mm and $B \times t_s = 800 \times 120$ mm, if $A_s = 5 \phi 16$ mm. Materials used are concrete $f_{cu}=30 \text{ N/mm}^2$ and st. grade 400/600.

Solution

a- case of $M_{cr}^{+ve}$

Example #2 - b

- case of $M_{cr}^{-ve}$

- Solve it yourself.

- (without calculations), For T-sections of same dimensions and steel area, Whichever is bigger $M_{cr}^{+ve}$ or $M_{cr}^{-ve}$ and why?
Lecture # 3

R.C. Fundamentals

Analysis of RC Sections under Flexure

➢ Stage I: Uncracked Section

➢ Stage II: Cracked Section in the working stage

➢ Stage III: Cracked Section at ultimate stage

Stage 2: Cracked section in Working stage

Analysis of cracked Section:

Steps

1- Determine the position of the neutral axis, N.A. \( z \)

2- Calculate the inertia of the cracked section \( I_{cr} \).

3- Calculate the working moment \( M_w \) using the maximum allowable stresses \( f_{c(all)} \) in conc. and \( f_{s(all)} \) for steel as follows:
Calculation of working moment $M_w$

1- for a singly reinforced section

Determination of the neutral axis and cracked transformed moment of inertia calculations

Calculation of working moment $M_w$

for a cracked Doubly Reinforced section

1- Determine the position of N.A. $(z)$: use $(n = 15)$

Taking moments of areas @ the N.A.

$$bZ\left(\frac{Z}{2}\right) - nA_s(d - z) = 0$$

$$\frac{b}{2}Z^2 + nA_sZ - nA_sd = 0$$

2- Calculate the inertia:

$$I_{cr} = \frac{bZ^3}{3} + nA_s(d - z)^2$$
3- Calculate the working moment $M_w$

$$M_{w1} = \frac{f_{c(all)} I_{cr}}{z}$$

$$M_{w2} = \frac{n I_{cr}}{(d - z)}$$

$M_w$ is the lesser of $M_{w1}, M_{w2}$

---

Calculation of working moment $M_w$

2- for a doubly reinforced section

Cracked section

Transformed section

Determination of the neutral axis and cracked transformed moment of inertia calculations
Calculation of working moment \( M_w \) for a cracked Doubly Reinforced section

1- Determine the position of N.A. \((z)\):
Taking moments of areas @ the N.A.

\[
\frac{bz}{2} + nA_s' (z - d') = nA_s (d - z)
\]

\[
\frac{b}{2} z^2 + n(A_s + A_s') z - n(A_s d + A_s' d') = 0
\]

2- Calculate the inertia:

\[
I_{cr} = \frac{b z^3}{3} + n A_s (d - z)^2 + n A_s' (z - d')^2
\]

3- Calculate the working moment \( M_w \)

\[
M_{w1} = \frac{f_{c(all)} I_{cr}}{z}
\]

\[
M_{w2} = \frac{f_{s(all)} I_{cr}}{n (d - z)}
\]

\( M_w \) is the lesser of \( M_{w1} \), \( M_{w2} \)
Working stresses used in Calculation of working moment (stage II - cracked section)

**Allowable stresses** in concrete and steel according to ECP Code 203 – Version 2007 From **Table 5 – 1:**

- Concrete in Case of - sections > 200 mm under flexure:

<table>
<thead>
<tr>
<th>$f_{cu}$</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{c, all - flexure}$</td>
<td>8</td>
<td>9.5</td>
<td>10.5</td>
</tr>
</tbody>
</table>

- Steel reinforcement

<table>
<thead>
<tr>
<th>$f_{y}$</th>
<th>240</th>
<th>360</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{s, all}$</td>
<td>140</td>
<td>200</td>
<td>220</td>
</tr>
</tbody>
</table>

**Table 5-1**

<table>
<thead>
<tr>
<th>جدول (5-1) إجهاد التشتاقل للخرسانة والصلب</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>أعمال الإجهادات</strong></td>
</tr>
<tr>
<td>-------------------------</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>10.5</td>
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<tr>
<td></td>
</tr>
<tr>
<td><strong>Steel reinforcement</strong></td>
</tr>
<tr>
<td>0.9</td>
</tr>
<tr>
<td>0.7</td>
</tr>
<tr>
<td>2.1</td>
</tr>
<tr>
<td>1.0</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>المصطلحات</strong></td>
</tr>
<tr>
<td>140</td>
</tr>
<tr>
<td>160</td>
</tr>
<tr>
<td>200</td>
</tr>
<tr>
<td>220</td>
</tr>
<tr>
<td>160</td>
</tr>
<tr>
<td>220</td>
</tr>
</tbody>
</table>

1. $f_{cu}$: Compressive strength of concrete
2. $f_{c, all - flexure}$: Compressive strength of concrete in all flexure
3. $f_{y}$: Tensile strength of steel
4. $f_{s, all}$: Tensile strength of steel in all sections
5. $q_e$: Tensile strength of concrete in flexure
6. $q_{ep}$: Tensile strength of concrete in all sections
7. $f_s$: Tensile strength of steel in all sections

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working stresses in conc. in Case of T- sections under flexure:

For □-section use:

\[ f_c = f_{c(\text{all})} = \text{allowable compression} \]

(bending) stress in concrete working stage (from table)\( f_{c(\text{all})} \)

For T- section use:

\[ f_{c(\text{av})} = \frac{2}{3} f_{c(\text{all})} \]

\[ M_{w1} = \frac{f_{c(\text{av})} I_{cr}}{z} \]

\( M_w \) is the least of \( M_{w1} \), \( M_{w2} \)

Calculation of working moment \( M^{+ve}_w \) for T-sec.

1- Determine the position of N.A) \( (z) \):

Taking moments of areas @ the N.A.

assume \( z < t_s \)

\[ B z \frac{z}{2} = n A_s (d - z) \]

\[ \frac{B}{2} z^2 + n A_s z - n A_s d = 0 \]

\[ z = \text{mm} \]
in case of \( z \leq t_s \)
the assumption is O.k.
Complete the solution, calculate \( I_{cr} \) then \( M_w \) is the lesser of \( M_{w1} \) \& \( M_{w2} \)

\[
\text{in case of } z > t_s \text{ then the assumption is wrong}
\]
Calculate another \( z \) Taking moments of areas @ the new N.A. then Complete the solution, calculate \( I_{cr} \) then \( M_w \) is the lesser of \( M_{w1} \) \& \( M_{w2} \)

- **Calculation of working moment*** \( M^{+ve}_w \) **for T-sec.**

*Case of \( z \leq t_s \)*

2- Calculate the inertia:

\[
I_{cr} = \frac{Bz^3}{3} + nA_s(d - z)^2
\]

3- Calculate the working moment \( M_w \)

\[
M_{w1} = \frac{2}{3} \frac{f_{c(all)} I_{cr}}{z}
\]

\[
M_{w2} = \frac{f_{s(all)} I_{cr}}{n(d - z)}
\]

\( M_w \) is the lesser of \( M_{w1} \), \( M_{w2} \)
- Calculation of working moment $M^\text{ve}_w$ for T-sec.

Case of $z > t_s$

1- Taking moments of areas @ the N.A.

$$\frac{b}{2}z^2 + (B-b)t_s(z - \frac{t_s}{2}) = nA_s(d - z)$$

$$\frac{b}{2}z^2 + [nA_s + (B-b)t_s]z - [nA_s d + (B-b)\frac{t_s}{2}] = 0$$

$$z = \text{mm}$$

2- Calculate the inertia:

$$I_{cr} = \frac{bz^3}{3} + (B-b)\frac{t_s^3}{12} + (B-b)t_s(z - \frac{t_s}{2})^2 + nA_s(d - z)^2$$

3- Calculate the working moment $M_w$

$$M_{w1} = \frac{2}{z} f_{c(all)} I_{cr}$$

$$M_{w2} = \frac{n f_{s(all)} I_{cr}}{(d - z)}$$

$M_w$ is the lesser of $M_{w1}$, $M_{w2}$

---

3- Calculation of working moment $M^\text{ve}_w$ for T-sec.

Solve as rectangular section as before
**Example - 3**

For the section shown

Calculate $M_w$

$f_{cw}=25 \ N/mm^2$

$f_y = 360 \ N/mm^2$

Solution

1- Determine the position of N.A. $(z)$: use $(n = 15)$

Taking moments of areas @ the N.A.

$$b \frac{z^2}{2} + nA_s' (z - d') = nA_s (d - z)$$

$$\frac{b}{2} z^2 + n(A_s + A'_s)z - n(A_s d + A'_s d') = 0$$

$A_s = 5 * 201 = 1005 \ mm^2$

$A'_s = 3 * 113 = 339 \ mm^2$

$125z^2 + 15(1005 + 339)z - 15[1005(750) + 339(50)] = 0$

$z^2 + 161.3z - 92484 = 0$

$z = 234 \ mm$

2- Calculate the inertia:

$$I_{cr} = \frac{b z^3}{3} + nA_s (d - z)^2 + nA'_s (z - d')^2$$

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\[ I_{cr} = \frac{bz^3}{3} + nA_s (d-z)^2 + nA_s \left( z - d' \right)^2 \]

\[ = \frac{250(234)^3}{3} + 15(1005)(750-234)^2 + 15(339)(234-50)^2 \]

\[ = 5.25 \times 10^9 \text{ mm}^4 \]

3- Calculate the working moment \( M_w \):

\[ M_{w1} = \frac{f_{c(alt)} I_{cr}}{z} = \frac{9.5(5.25 \times 10^9)}{234} \]

\[ = 213.1 \times 10^6 \text{ N.mm} = 213.1 \text{ kN.m} \]

\[ f_{s(alt)} I_{cr} = \frac{200}{15} (5.25 \times 10^9) \]

\[ = 135.7 \text{ kN.m} \]

\( M_w \) is the lesser of \( M_{w1}, M_{w2} \)

\( M_w = 135.7 \text{ kN.m} \)

Example - 4

**Calculation of working moment \( M_w \) – T-section**

For the section shown

Calculate \( M_w \)

\( f_{cu} = 25 \text{ N/mm}^2 \)

\( f_y = 360 \text{ N/mm}^2 \)

\[ b = 200 \text{ mm} \]

\[ h = 700 \text{ mm} \]
**Solution**

1. Assume \( z < t_s \)

   \[ t_s = 120 \text{ mm} \]

   1- Determine the position of N.A. \((z)\):

   Taking moments of areas @ the N.A.

   \[
   Bz \cdot \frac{z}{2} = nA_s \left( d - z \right)
   \]

   \[
   \frac{B}{2} z^2 + nA_s z - nA_s d = 0
   \]

   \[
   400z^2 + 15(1524) z - 15(1524)(630) = 0
   \]

   \[
   z^2 + 57.15 z - 36004.5 = 0
   \]

   \[
   z = 163.3 \text{ mm} > t_s = 120 \text{ mm}
   \]

   The assumption is wrong

   \( z > t_s \)

   Taking moments of areas @ the new N.A.

   \[
   b_z \cdot \frac{z}{2} + (B-b)t_s \left( z - \frac{t_s}{2} \right) = nA_s \left( d - z \right)
   \]

   \[
   \frac{b}{2} z^2 + [nA_s + (B-b)t_s] z - [nA_s d + (B-b) \frac{t_s}{2}] = 0
   \]

   \[
   100z^2 + [15(1524) + [(800 - 200)(120)] z
   \]

   \[
   - [15(1524)(630) + (800 - 200) \frac{120^2}{2}] = 0
   \]

   \[
   z^2 + 948.6 z - 187218 = 0
   \]

   \[
   z = 167.7 \text{ mm}
   \]
2- Calculate the inertia $I_{cr}$:

\[
I_{cr} = \frac{bz^3}{3} + (B-b)\frac{t_s^3}{12} + (B-b)t_s\left(z - \frac{t_s}{2}\right)^2 + nA_s\left(d - z\right)^2
\]

\[
I_{cr} = 200\left(167.7\right)^3 + \frac{(800-200)(120)^3}{12} + (800-200)120\left(167.7 - 60\right)^2 + 15(1524)(630 - 167.7)^2
\]

\[
= 6.1 \times 10^9 \quad \text{mm}^4
\]

3- Calculate the working moment $M_w$:

\[
M_{w1} = \frac{f_{c(\text{av})}I_{cr}}{z}
\]

\[
M_{w1} = \frac{2}{3} \left(9.5\right)\left(6.1 \times 10^9\right)
\]

\[
= \frac{2}{3} \times 167.7 \times 10^6
\]

\[
= 230.4 \quad \text{kN.m}
\]

\[
M_{w2} = \frac{n}{(d - z)} \frac{f_{s(\text{all})}I_{cr}}{15}
\]

\[
= \frac{220}{15} \left(6.1 \times 10^9\right)
\]

\[
= 193.5 \quad \text{kN.m}
\]

$M_w$ is the lesser of $M_{w1}$, $M_{w2}$

$M_w = 193.5 \quad \text{kN.m}$