BAS007 Mechanics 2

Second Semester 2015-2016
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Lecture 4

Kinetics of a Particle
Force and Acceleration

OBJECTIVES

➢ To state Newton's Second Law of Motion and to define mass and weight.

➢ To analyze the accelerated motion of a particle using the equation of motion with different coordinate systems.

➢ To investigate central-force motion and apply it to problems in space mechanics.
Newton's Second Law of Motion

Kinetics
A branch of dynamics that deals with the relationship between the change in motion of a body and the forces that cause this change

Newton's second law
“when an unbalanced force acts on a particle, the particle will accelerate in the direction of the force with a magnitude that is proportional to the force”

\[ F \alpha a \]
\[ F = m a \]
Newton's Law of Gravitational Attraction.

The law governing the mutual attraction between any two particles. In mathematical form this law can be expressed as:

$$ F = G \frac{m_1 m_2}{r^2} $$

where:

- $F = \text{force of attraction between the two particles}$
- $G = \text{universal constant of gravitation; according to experimental evidence, } G = 66.73 \times 10^{-12} \, m^3/kg \cdot s^2$
- $m_1, m_2 = \text{mass of each of the two particles}$
- $r = \text{distance between the centers of the two particles}$
Equation of Motion for a System of Particles

\[ \sum F_i = \sum m_i a_i \]
Equations of Motion: Rectangular Coordinates

\[ \sum F = ma \]
\[ \sum F = \sum F_x i + \sum F_y j + \sum F_z k \]
\[ \sum F = m (a_x i + a_y j + a_z k) \]
Procedure for Analysis

1. *Free-Body Diagram*

- Select the inertial coordinate system. Most often, rectangular or $x, y, z$ coordinates are chosen to analyze problems for which the particle has rectilinear motion.

- Once the coordinates are established, draw the particle's free body diagram. Drawing this diagram is very important since it provides a graphical representation that accounts for all the forces which act on the particle, and thereby makes it possible to resolve these forces into their $x, y, z$ components.

- The direction and sense of the particle's acceleration $a$ should also be established. If the sense is unknown, for mathematical convenience assume that the sense of each acceleration component acts in the same direction as its positive inertial coordinate axis.

- Identify the unknowns in the problem.
Procedure for Analysis

2. Equation of motion

- If the forces can be resolved directly from the free-body diagram, apply the equations of motion in their scalar component form.

- If the geometry of the problem appears complicated, which often occurs in three dimensions, Cartesian vector analysis can be used for the solution.

- **Friction.** If a moving particle contacts a rough surface, it may be necessary to use the frictional equation, which relates the frictional and normal forces $F_f$ and $N$ acting at the surface of contact by using the coefficient of kinetic friction, i.e., $F_f = \mu_k \; N$. Remember that $F_f$ always acts on the free-body diagram such that it opposes the motion of the particle relative to the surface it contacts. If the particle is on the verge of relative motion, then the coefficient of static friction should be used.

- **Spring.** If the particle is connected to an elastic spring having negligible mass, the spring force $F_s$ can be related to the deformation of the spring by the equation $(F_s=K \; S)$. Here $K$ is the spring's stiffness measured as a force per unit length, and $S$ is the stretch or compression defined as the difference between the deformed length $l$ and the undeformed length $l_0$, i.e., $S = l - l_0$. 
Procedure for Analysis

3. Kinematics

- If the velocity or position of the particle is to be found, it will be necessary to apply the necessary kinematic equations once the particle's acceleration is determined from $\Sigma F = ma$.

- If acceleration is a function of time, use $a = dv/dt$ and $v = ds/dt$ which, by integration, yield the particle's velocity and position, respectively.

- If acceleration is a function of displacement, integrate $a \, ds = v \, dv$ to obtain the velocity as a function of position.

- If acceleration is constant, use $V = V_o + at$, $S = S_o + V_o t + 0.5at^2$, $V^2 = V_o^2 + 2a(S - S_o)$ to determine the velocity or position of the particle.

- Make sure the positive inertial coordinate directions used for writing the kinematic equations are the same as those used for writing the equations of motion; otherwise, simultaneous solution of the equations will result in errors.

- If the solution for an unknown vector component yields a negative scalar, it indicates that the component acts in the direction opposite to that which was assumed.
The 50-kg crate shown in Figure rests on a horizontal surface for which the coefficient of kinetic friction is $\mu_k = 0.3$. If the crate is subjected to a 400-N towing force as shown, determine the velocity of the crate in 3 s starting from rest.

1. **Free Body Diagram**

$$W = mg = 490.5 \text{ N}$$

![Free Body Diagram](image)

2. **Equation of motion**

$$\sum F_x = ma_x; \quad 400 \cos 30^\circ - 0.3N_C = 50a$$  \hspace{1cm} (1)

$$\sum F_y = ma_y; \quad N_C - 490.5 + 400 \sin 30^\circ = 0$$  \hspace{1cm} (2)

$N_C = 290.5 \text{ N}$ \hspace{1cm} $a = 5.185 \text{ m/s}^2$

3. **Kinematics**

$$v = v_0 + at = 0 + 5.185(3)$$

$$= 15.6 \text{ m/s} \rightarrow$$
A smooth 2-kg collar C, is attached to a spring having a stiffness \( k = 3 \text{ N/m} \) and an unstretched length of 0.75 m. If the collar is released from rest at A, determine its acceleration and the normal force of the rod on the collar at the instant \( y = 1 \text{ m} \)

**1. Free Body Diagram**

![Free Body Diagram]

**2. Equation of motion**

\[
\begin{align*}
\Sigma F_x &= ma_x; \\
-N_C + F_s \cos \theta &= 0 \\
\Sigma F_y &= ma_y; \\
19.62 - F_s \sin \theta &= 2a
\end{align*}
\]

\[s = CB - AB = \sqrt{y^2 + (0.75)^2} - 0.75.\]

\[F_s = ks\]

\[\theta = 53.1^\circ\]

\[N_C = 0.900 \text{ N}\]

\[a = 9.21 \text{ m/s}^2\]
The 100-kg block A is released from rest. If the masses of the pulleys and the cord are neglected, determine the speed of the 20-kg block B in 2 s.

1. **Free Body Diagram**

2. **Equation of motion**

   Block A,
   \[ +\sum F_y = ma_y; \]
   \[ 981 - 2T = 100a_A \]

   Block B,
   \[ +\sum F_y = ma_y; \]
   \[ 196.2 - T = 20a_B \]

3. **Kinematics**

   \[ 2s_A + s_B = l \]
   \[ 2a_A = -a_B \]

   \[ T = 327.0 \text{ N} \]
   \[ a_A = 3.27 \text{ m/s}^2 \]
   \[ a_B = -6.54 \text{ m/s}^2 \]

   \[ v = v_0 + a_Bt \]
   \[ = 0 + (-6.54)(2) \]
   \[ = -13.1 \text{ m/s} \]
Important notices

• The equation of motion states that the unbalanced force on a particle causes it to accelerate.

• Mass is a property of matter that provides a quantitative measure of its resistance to a change in velocity. It is an absolute quantity and so it does not change from one location to another.

• Weight is a force that is caused by the earth's gravitation. It is not absolute; rather it depends on the altitude of the mass from the earth's surface.