Chapter 2
Satellite Orbits

The orbital locations of the spacecraft in a communications satellite system play a major role in determining the coverage and operational characteristics of the services provided by that system.
The same laws of motion that control the motions of the planets around the sun govern artificial earth satellites that orbit the earth.

Figure 2.1 shows a simplified picture of the forces acting on an orbiting satellite.

The gravitational force

angular velocity force
The gravitational force, $F_{in}$, and the angular velocity force, $F_{out}$, can be represented as

$$F_{in} = m \left( \frac{\mu}{r^2} \right) \quad F_{out} = m \left( \frac{v^2}{r} \right)$$

Where
- $m =$ satellite mass;
- $v =$ satellite velocity in the plane of orbit;
- $r =$ distance from the center of the earth (orbit radius);
- $\mu =$ Kepler’s Constant (or Geocentric Gravitational Constant) $= 3.986004 \times 10^5$ km$^3$/s$^2$. 
Note that for $F_{in} = F_{out}$

$$v = \left( \frac{\mu}{r} \right)^{\frac{1}{2}}$$

- This result gives the velocity required to maintain a satellite at the orbit radius $r$. 
2.1 Kepler’s Laws

*Kepler’s First Law*, as it applies to artificial satellite orbits, can be simply stated as follows:

‘the path followed by a satellite around the earth will be an ellipse, with the center of mass of earth as one of the two foci of the ellipse.’
Kepler’s Second Law can likewise be simply stated as follows: ‘for equal time intervals, the satellite sweeps out equal areas in the orbital plane.’ Figure 2.3 demonstrates this concept.
The shaded area $A_1$ shows the area swept out in the orbital plane by the orbiting satellite in a one hour time period at a location near the earth.

Kepler’s second law states that the area swept out by any other one hour time period in the orbit will also sweep out an area equal to $A_1$. 
Kepler’s Third Law is as follows: ‘the square of the periodic time of orbit is proportional to the cube of the mean distance between the two bodies. This is quantified as follows:

\[ T^2 = \left[ \frac{4\pi^2}{\mu} \right] a^3 \]

where
- \( T \) = orbital period in s;
- \( a \) = distance between the two bodies, in km;
- \( \mu \) = Kepler’s Constant = \( 3.986004 \times 10^5 \) km\(^3\)/s\(^2\).

If the orbit is circular, then \( a = r \), and

\[ r = \left[ \frac{\mu}{4\pi^2} \right]^{\frac{1}{3}} T^2 \]
This demonstrates an important result:
Orbit Radius = [Constant] \times (Orbit Period)^{2/3}

*Under this condition, a specific orbit period is determined only by proper selection of the orbit radius.

* This allows the satellite designer to select orbit periods that best meet particular application requirements by locating the satellite at the proper orbit altitude.

*The altitudes required to obtain a specific number of repeatable ground traces with a circular orbit are listed in Table
<table>
<thead>
<tr>
<th>Revolutions/day</th>
<th>Nominal period (hours)</th>
<th>Nominal altitude (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24</td>
<td>36 000</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>20 200</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>13 900</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>10 400</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>6 400</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>4 200</td>
</tr>
</tbody>
</table>
2.2 Orbital Parameters

Figure 2.4 Earth-orbiting satellite parameters

1- **Apogee** – the point farthest from earth.

2- **Perigee** – the point of closest approach to earth.

3- **Line of Apsides** – the line joining the perigee and apogee through the center of the earth.

4- **Ascending Node** – the point where the orbit crosses the equatorial plane, going from south to north.
5- **Descending Node** – the point where the orbit crosses the equatorial plane, going from north to south.

6- **Line of Nodes** – the line joining the ascending and descending nodes through the center of the earth.

7- **Argument of Perigee**, \( \phi \) – the angle from ascending node to perigee, measured in the orbital plane.

8- **Right Ascension of the Ascending Node**, \( \psi \) – the angle measured eastward, in the equatorial plane, from the line to the first point of Aries (Y) to the ascending node.

The **eccentricity** \( e \) is a measure of the ‘circularity’ of the orbit. It is determined from

\[
e = \frac{r_a - r_p}{r_a + r_p}
\]
where \( e \) = the eccentricity of the orbit; 
\( r_a \) = the distance from the center of the earth to the apogee point; 
\( r_p \) = the distance from the center of the earth to the perigee point.

The higher the eccentricity, \( e \) the ‘flatter’ the ellipse. A circular orbit is the special case of an ellipse with equal major and minor axes (zero eccentricity). That is:

Elliptical Orbit \( 0 < e < 1 \)

Circular Orbit \( e = 0 \)

‘flatter \( e \) is very large
The *inclination angle* $\theta$, is the angle between the orbital plane and the earth’s equatorial plane. A satellite that is in an orbit with some inclination angle is in an *inclined orbit*.

A satellite that is in orbit in the equatorial plane (inclination angle $= 0^\circ$) is in an *equatorial orbit*.

A satellite that has an inclination angle of $90^\circ$ is in a *polar orbit*.

The orbit may be *elliptical* or *circular*, depending on the orbital velocity and direction of motion imparted to the satellite on insertion into orbit.
Figure 2.5 shows another important characteristic of satellite orbits.

An orbit in which the satellite moves in the same direction as the earth’s rotation is called a prograde orbit.

The inclination angle of a prograde orbit is between $0^\circ$ and $90^\circ$.

A satellite in a retrograde orbit (عكسي) moves in a direction opposite (counter to) the earth’s rotation, with an inclination angle between $90^\circ$ and $180^\circ$.

Most satellites are launched in a prograde orbit, because the earth’s rotational velocity enhances the satellite’s orbital velocity, reducing the amount of energy required to launch and place the satellite in orbit.
Figure 2.5  Prograde and retrograde orbits
Orbital elements

The minimum number of parameters required is six

1- Eccentricity (e)
2- Semi-Major Axis; \((r_a, r_p)\)
3- Time of Perigee (الانخفاض); \(T\)
4- Right Ascension of Ascending Node (الصعود)
5- Inclination Angle; \(\theta\)
6- Argument of Perigee (W)
Sidereal time

Satellite orbits coordinates are specified in *sidereal time*ِالوقت الفلكي* rather than in solar time.

Sidereal day: one complete rotation of the earth relative to the fixed stars.

1 mean Solar day = 1.002738 mean Sidereal days
1 mean Sidereal day = 0.9972696 mean Solar days
2.3.1 Geostationary Orbit GEO

**Orbit radius, \( r_S \)**

From Kepler’s third law, the orbit radius for the GEO, \( r_S \), is found as

\[
\begin{align*}
\frac{r_S}{r_S} &= \left[ \frac{\mu}{4 \pi^2} \right]^{\frac{1}{3}} T^2 \\
&= \left[ \frac{3.986004 \times 10^5}{4 \pi^2} \right]^{\frac{1}{3}} (86\,164.09)^{\frac{2}{3}} \\
&= 42\,164.17 \text{ km}
\end{align*}
\]

where \( T = 1 \) mean sidereal day = 86 164.09 s.

=23x60x60+56x60+4=86 164.09 s.(23 hour+56 min+4 sec)
**Geostationary orbit height** $h_S$

The geostationary height (altitude above the earth’s surface), $h_S$, is then

$$h_S = r_S - r_E$$

$$= 42164 - 6378$$

$$= 35786 \text{ km}$$

$r_S$ orbit radius

where $r_E = \text{equatorial earth radius} = 6378 \text{ km}$. The value of $h_S$ is often rounded to 36 000 km for use in orbital calculations.

The geostationary orbit is an ideal orbit that cannot be achieved for real artificial satellites because there are many other forces besides the earth’s gravity acting on the satellite.
A typical GEO orbit in use today would have an inclination angle slightly greater than 0 and possibly an eccentricity that also exceeds 0.

The ‘real world’ GEO orbit that results is often referred to as a *geosynchronous earth orbit* (GSO) to differentiate it from the ideal geostationary orbit.

Figure 2.7 shows the basic elements of the geosynchronous earth orbit as it applies to satellite operations.
23 000 n mi  
(36 000 km)  
circular orbit in equatorial plane

- most common
- fixed slant paths
- little or no ground station tracking required
- 2 or 3 satellites for global coverage (except for poles)

Geostationary (GEO) – ideal orbit (inclination = 0°)
Geosynchronous (GSO) – all real orbits (inclination ≠ 0°)

Figure 2.7  GSO – Geosynchronous earth orbit
2.3.2 Low Earth Orbit

*It operate well below the geostationary altitude, typically at altitudes from 160 to 2500 km, and in near circular orbits.

*The low earth orbit satellite has several characteristics that can be advantageous for communications applications, as summarized on Figure 2.8.
- requires earth terminal tracking
  – approx. 8 to 10 minutes per pass for an earth terminal
  – requires multiple satellites (12, 24, 66, . . . ) for global coverage
  – popular for mobile satellite communications applications

Figure 2.8 LEO – Low earth orbit
2.4 Geometry of Geosynchronous (GSO) Links

The GSO is the dominant orbit used for communications satellites.

The parameters required to define the GSO parameters that are used to evaluate satellite link performance and design.

d = range (distance) from the earth station (ES) to the satellite, in km
\( \phi_z \) = azimuth angle from the ES to the satellite, in degrees
\( \theta \) = elevation angle from the ES to the satellite, in degrees

The azimuth and elevation angles are referred to as the look angles for the ES to the satellite.
the geometry and definitions of the look angles with respect to the earth station reference.

\[ \phi_z = \text{azimuth angle to satellite} \]
\[ \theta = \text{elevation angle to satellite} \]

**Figure 2.11** GSO look angles to satellite
The input parameters required to determine the GSO parameters are:

- $l_E =$ earth station longitude, in degrees
- $l_S =$ satellite longitude, in degrees
- $L_E =$ earth station latitude, in degrees
- $L_S =$ satellite latitude in degrees (assumed to be 0, i.e., inclination angle = 0)
- $H =$ earth station altitude above sea level, in km

The point on the earth’s equator at the satellite longitude is called the *subsatellite point* (SS).

Figure 2.12 clarifies the definition of earth station altitude.
Longitude and latitude sign values are based on the sign convention shown in Figure 2.13. Longitudes east of the Greenwich Meridian and latitudes north of the equator are positive.

Figure 2.13 sign convention for Longitude and latitude
Additional parameters required for the calculations are:

- Equatorial Radius: \( r_e = 6378.14 \text{ km} \)
- Geostationary Radius: \( r_S = 42164.17 \text{ km} \)
- Geostationary Height (Altitude): \( h_{GSO} = r_S - r_e = 35786 \text{ km} \)
- Eccentricity of the earth: \( e_e = 0.08182 \)
- Differential longitude, \( B \), defined as the difference between the earth station and satellite longitudes:
  \[ B = l_E - l_S \text{ in degree} \]

Example, for an earth station located in Washington, DC, at the longitude of \( 77^\circ W \), and a satellite located at a longitude of \( 110^\circ W \):

\[ B = (-77) - (-110) = +33^\circ \]
2.4.1 Range to Satellite \( R \)
The determination of the range to the satellite from the earth station requires the radius of the earth at the earth station latitude and longitude, \( R \). It is found as

\[
R = \sqrt{l^2 + z^2}
\]

\[
l = \left( \frac{r_e}{\sqrt{1 - e_e^2 \sin^2(L_E)}} + H \right) \cos(L_E)
\]

\[
z = \left( \frac{r_e (1 - e_e^2)}{\sqrt{1 - e_e^2 \sin^2(L_E)}} + H \right) \sin(L_E)
\]

An intermediate angle, \( \varphi_E \) is,

\[
\Phi_E = \tan^{-1}\left( \frac{z}{l} \right)
\]

The range \( d \) is then found from

\[
d = \sqrt{R^2 + r_s^2 - 2 R r_s \cos(\Psi_E) \cos(B)}
\]
2.4.2 Elevation Angle to Satellite

The elevation angle from the earth station to the satellite, $\theta$, is determined from

$$\theta = \cos^{-1} \left( \frac{r_e + h_{GSO}}{d} \sqrt{1 - \cos^2(B) \cos^2(L_E)} \right)$$

where

- $r_e =$ equatorial radius $= 6378.14$ km;
- $h_{GSO} =$ geostationary altitude $= 35786$ km;
- $d =$ range, in km;
- $B =$ differential longitude, in degrees;
- $L_E =$ Earth Station latitude, in degrees.
2.4.3 Azimuth Angle to Satellite

The final parameter of interest is the earth station azimuth angle to the satellite. First, an intermediate angle $A_i$ is found from

$$A_i = \sin^{-1}\left(\frac{\sin(|B|)}{\sin(\beta)}\right)$$

where $|B|$ is the absolute value of the differential longitude

$$|B| = |L_E - L_S|$$

$$\beta = \cos^{-1}[\cos(B)\cos(L_E)]$$

The azimuth angle $\varphi_z$ is determined from the intermediate angle $A_i$ from one of four possible conditions, based on the relative location of the earth station and the subsatellite point on the earth surface
<table>
<thead>
<tr>
<th>Condition*</th>
<th>( \varphi_z = )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS point is NE of ES</td>
<td>( A_i )</td>
</tr>
<tr>
<td>SS point is NW of ES</td>
<td>( 360 - A_i )</td>
</tr>
<tr>
<td>SS point is SE of ES</td>
<td>( 180 - A_i )</td>
</tr>
<tr>
<td>SS point is SW of ES</td>
<td>( 180 + A_i )</td>
</tr>
</tbody>
</table>

**NE** North east  
**NW** North west  
**SE** South east  
**SW** South west  
**SS** Subsatellite point  
**ES** Earth station
2.4.4 Sample Calculation
This section presents a sample calculation for the determination of the GSO parameters described above.

Consider an earth station located in Washington, DC, and a GSO satellite located at 97° W. The input parameters, using the sign conventions of Figure 2.13 are:

**Earth Station: Washington, DC**
- Latitude: \( L_E = 39^\circ \text{ N} = +39 \)
- Longitude: \( l_E = 77^\circ \text{ W} = -77 \)
- Altitude: \( H = 0 \text{ km} \)

**Satellite:**
- Latitude: \( L_S = 0^\circ \) (inclination angle = 0)
- Longitude: \( l_S = 97^\circ \text{ W} = -97 \)
Find the range, \( d \), the elevation angle, \( \theta \), and the azimuth angle, \( \phi_z \), to the satellite.

Step 1) Determine the differential longitude, \( B \), (Equation (2.10)):

\[
B = l_E - l_s = (-77) - (-97) = +20
\]

Step 2) Determine the earth radius at the earth station, \( R \), for the calculation of the range (Equations (2.11) to (2.14)):

\[
l = \left( \frac{r_e}{\sqrt{1 - e_e^2 \sin^2(L_E)}} + H \right) \cos(L_E)
\]

\[
= \left( \frac{6378.13}{\sqrt{1 - (0.08182)^2 \sin^2(39^\circ)}} + 0 \right) \cos(39^\circ) = 4963.33 \text{ km}
\]

\[
z = \left( \frac{r_e (1 - e_e^2)}{\sqrt{1 - e_e^2 \sin^2(L_E)}} + H \right) \sin(L_E)
\]
\[
= \left( \frac{6378.14(1 - 0.08182^2)}{\sqrt{1 - 0.08182^2 \sin^2(39^\circ)}} + 0 \right) \sin(39^\circ) = 3992.32 \text{ km}
\]

\[
\Psi_E = \tan^{-1} \left( \frac{Z}{l} \right) = \tan^{-1} \left( \frac{3992.32}{4963.33} \right) = 38.81^\circ
\]

\[
R = \sqrt{l^2 + z^2} = \sqrt{4963.33^2 + 3992.32^2} = 6369.7 \text{ km}
\]

Step 3) Determine the range, \( d \), (Equation (2.15)):

\[
d = \sqrt{R^2 + r_s^2 - 2 \cdot R \cdot r_s \cos(\Psi_E) \cos(B)}
\]

\[
= \sqrt{6369.7^2 + 42164^2 - 2 \times 6369.7 \times 42164 \times \cos(38.81^\circ) \times \cos(20^\circ)}
\]

\[
d = 37750 \text{ km}
\]
Step 4) Determine the elevation angle, $\theta$, (Equation (2.16)):

$$
\theta = \cos^{-1} \left( \frac{r_e + h_{GSO}}{d} \sqrt{1 - \cos^2(B) \cos^2(L_e)} \right)
$$

$$
= \cos^{-1} \left( \frac{6378.14 + 35786}{37750} \sqrt{1 - \cos^2(20^\circ) \cos^2(39^\circ)} \right)
$$

$$
\theta = 40.27^\circ
$$

Step 5) Determine the intermediate angle, $A_i$, (Equation (2.17)):

$$
\beta = \cos^{-1} [\cos(B) \cos(L_e)]
$$

$$
= \cos^{-1} [\cos(20) \cos(39)]
$$

$$
= 43.09^\circ
$$

$$
A_i = \sin^{-1} \left( \frac{\sin(|B|)}{\sin(\beta)} \right)
$$
\[
= \sin^{-1}\left(\frac{\sin(20)}{\sin(43.09)}\right)
\]
\[
= 30.04^\circ
\]

Step 6) Determine the azimuth angle, \( \varphi_z \), from the intermediate angle, \( A_i \), (see Figure 2.14 and Table 2.2).

Since the subsatellite point SS is southwest of the earth station ES, condition (d) holds and

\[
\varphi_z = 180 + A_i
\]
\[
= 180 + 30.04
\]

Summary: The orbital parameters for the Washington, DC, ground station are:

d = 37,750 \text{ km}
\theta = 40.27^\circ
\varphi_z = 210.04^\circ