Chapter 1
Series AC Circuit

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1 Purely resistive a.c. circuit

**Circuit diagram**

**Phasor diagram**

**Current and voltage waveforms**
1.2 Purely inductive a.c. circuit

- Circuit diagram
- Phasor diagram
- Current and voltage waveforms

$I_L$ lags $V_L$ by $90^\circ$
\[ X_L = \frac{V_L}{I_L} = 2\pi fL \ \Omega \]

**Problem 1.** (a) Calculate the reactance of a coil of inductance 0.32 H when it is connected to a 50 Hz supply. (b) A coil has a reactance of 124 \( \Omega \) in a circuit with a supply of frequency 5 kHz. Determine the inductance of the coil.

(a) Inductive reactance, \( X_L = 2\pi fL = 2\pi (50)(0.32) \)

\[ = 100.5 \ \Omega \]

(b) Since \( X_L = 2\pi fL \), inductance

\[ L = \frac{X_L}{2\pi f} = \frac{124}{2\pi (5000)} \ H \]

\[ = 3.95 \text{ mH} \]
3 Purely capacitive a.c. circuit

\[ X_C = \frac{V_C}{I_C} = \frac{1}{2\pi fC} \Omega \]
Problem 3. Determine the capacitive reactance of a capacitor of 10μF when connected to a circuit of frequency (a) 50Hz (b) 20kHz.

(a) Capacitive reactance $X_C = \frac{1}{2\pi fC}$

\[
= \frac{1}{2\pi (50)(10 \times 10^{-6})} \\
= \frac{10^6}{2\pi (50)(10)} \\
= 318.3 \Omega
\]

(b) $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi (20 \times 10^3)(10 \times 10^{-6})}$

\[
= \frac{10^6}{2\pi (20 \times 10^3)(10)} \\
= 0.796 \Omega
\]
4 \( R-L \) series a.c. circuit
The ‘voltage triangle’ is derived. For the $R–L$ circuit:

$$V = \sqrt{V_R^2 + V_L^2} \quad \text{(by Pythagoras’ theorem)}$$

and $$\tan \phi = \frac{V_L}{V_R} \quad \text{(by trigonometric ratios)}$$

In an a.c. circuit, the ratio $\frac{\text{applied voltage } V}{\text{current } I}$ is called the impedance $Z$, i.e.

$$Z = \frac{V}{I} \Omega$$
Problem 6. In a series $R$–$L$ circuit the p.d. across the resistance $R$ is 12V and the p.d. across the inductance $L$ is 5V. Find the supply voltage and the phase angle between current and voltage.

For the $R$–$L$ circuit: $Z = \sqrt{R^2 + X_L^2}$

$$\tan \phi = \frac{X_L}{R}, \sin \phi = \frac{X_L}{Z} \text{ and } \cos \phi = \frac{R}{Z}$$
supply voltage $\bar{V} = \sqrt{12^2 + 5^2}$ i.e. $V = 13V$

(Note that in a.c. circuits, the supply voltage is not the arithmetic sum of the p.d.’s across components. It is, in fact, the **phasor sum**.)

$$\tan \phi = \frac{V_L}{V_R} = \frac{5}{12}, \text{ from which } \phi = \tan^{-1} \left( \frac{5}{12} \right)$$

$$= 22.62^\circ \text{ lagging}$$
Problem 8

A coil takes a current of 2A from a 12V d.c. supply. When connected to a 240V, 50Hz supply the current is 20A. Calculate the resistance, impedance, inductive reactance and inductance of the coil.
Resistance \( R = \frac{\text{d.c. voltage}}{\text{d.c. current}} = \frac{12}{2} = 6 \Omega \)

Impedance \( Z = \frac{\text{a.c. voltage}}{\text{a.c. current}} = \frac{240}{20} = 12 \Omega \)

Since \( Z = \sqrt{R^2 + X_L^2} \), inductive reactance,

\[
X_L = \sqrt{(Z^2 - R^2)}
\]

\[
= \sqrt{(12^2 - 6^2)}
\]

\[
= 10.39 \Omega
\]

Since \( X_L = 2\pi fL \), inductance \( L = \frac{X_L}{2\pi f} = \frac{10.39}{2\pi (50)} \)

\[
= 33.1 \text{ mH}
\]
Problem 11

A pure inductance of 1.273mH is connected in series with a pure resistance of 30. If the frequency of the sinusoidal supply is 5kHz and the p.d. across the 30 resistor is 6V, determine the value of the supply voltage and the voltage across the 1.273mH inductance. Draw the phasor diagram.
Supply voltage, $V = IZ$

Current $I = \frac{V_R}{R} = \frac{6}{30} = 0.20 \text{ A}$

Inductive reactance $X_L = 2\pi fL$

$= 2\pi (5 \times 10^3)(1.273 \times 10^{-3})$

$= 40 \Omega$
Impedance, \( Z = \sqrt{R^2 + X_L^2} = \sqrt{30^2 + 40^2} = 50 \Omega \)

Supply voltage \( V = IZ = (0.20)(50) = 10 \text{ V} \)

Voltage across the 1.273 mH inductance, \( V_L = IX_L \)
\[
= (0.2)(40)
\]
\[
= 8 \text{ V}
\]
5 \( R-C \) series a.c. circuit
From the phasor diagram of Figure 10

\[ V = \sqrt{V_R^2 + V_C^2} \quad (\text{by Pythagoras' theorem}) \]

and \[ \tan \alpha = \frac{V_C}{V_R} \quad (\text{by trigonometric ratios}) \]

For the \( R-C \) circuit: \[ Z = \sqrt{R^2 + X_C^2} \]

\[ \tan \alpha = \frac{X_C}{R}, \quad \sin \alpha = \frac{X_C}{Z} \quad \text{and} \quad \cos \alpha = \frac{R}{Z} \]
Problem 13

A resistor of 25 is connected in series with a capacitor of 45\,\mu\text{F}. Calculate (a) the impedance, and (b) the current taken from a 240\,\text{V}, 50\,\text{Hz} supply. Find also the phase angle between the supply voltage and the current.
\[ R = 25 \, \Omega; \quad C = 45 \, \mu F = 45 \times 10^{-6} \, F; \quad V = 240 \, V; \]
\[ f = 50 \, \text{Hz} \]

The circuit diagram is as shown in Figure 15.10

Capacitive reactance, \( X_C = \frac{1}{2\pi f C} \)

\[
= \frac{1}{2\pi (50)(45 \times 10^{-6})}
= 70.74 \, \Omega
\]
(a) Impedance \( Z = \sqrt{R^2 + X_C^2} = \sqrt{(25)^2 + (70.74)^2} \)
\[= 75.03 \Omega \]

(b) Current \( I = \frac{V}{Z} = \frac{240}{75.03} = 3.20 \text{ A} \)
Phase angle between the supply voltage and current,
\[\alpha = \tan^{-1}\left(\frac{X_C}{R}\right)\]
hence \(\alpha = \tan^{-1}\left(\frac{70.74}{25}\right) = 70.54^\circ \text{ leading} \)
Problem 14

A capacitor $C$ is connected in series with a 40 resistor across a supply of frequency 60Hz. A current of 3 A flows and the circuit impedance is 50. Calculate: (a) the value of capacitance, $C$, (b) the supply voltage, (c) the phase angle between the supply voltage and current, (d) the p.d. across the resistor, and (e) the p.d. across the capacitor. Draw the phasor diagram.
(a) Impedance \( Z = \sqrt{R^2 + X_C^2} \)

Hence \( X_C = \sqrt{Z^2 - R^2} = \sqrt{50^2 - 40^2} = 30 \, \Omega \)

\[
X_C = \frac{1}{2\pi f C}
\]

hence \( C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi (60)30} \) F

\[
= 88.42 \mu F
\]

(b) Since \( Z = \frac{V}{I} \) then \( V = IZ = (3)(50) = 150 \, V \)
(c) Phase angle, \( \alpha = \tan^{-1} \frac{X_C}{R} = \tan^{-1} \left( \frac{30}{40} \right) = 36.87^\circ \text{ leading} \)

(d) P.d. across resistor, \( V_R = IR = (3)(40) = 120 \text{ V} \)

(e) P.d. across capacitor, \( V_C = IX_C = (3)(30) = 90 \text{ V} \)

The phasor diagram is shown in Figure 11, where the supply voltage \( V \) is the phasor sum of \( V_R \) and \( V_C \).

\[ V_R = 120 \text{ V} \quad I = 3 \text{ A} \]

\[ V_C = 90 \text{ V} \quad V = 150 \text{ V} \]

Phasor diagram
6 $R–L–C$ series a.c. circuit

When $X_L > X_C$ (Figure 12(b)):

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

and $\tan\phi = \frac{(X_L - X_C)}{R}$

When $X_C > X_L$ (Figure 12(c)):

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

and $\tan\alpha = \frac{(X_C - X_L)}{R}$
A coil of resistance 5 and inductance 120mH in series with a 100μF capacitor, is connected to a 300V, 50Hz supply. Calculate (a) the current flowing, (b) the phase difference between the supply voltage and current, (c) the voltage across the coil and (d) the voltage across the capacitor.
\[ X_L = 2\pi fL = 2\pi (50)(120 \times 10^{-3}) = 37.70 \, \Omega \]

\[ X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi (50)(100 \times 10^{-6})} = 31.83 \, \Omega \]

Since \( X_L \) is greater than \( X_C \) the circuit is inductive.

\[ X_L - X_C = 37.70 - 31.83 = 5.87 \, \Omega \]

Impedance \( Z = \sqrt{R^2 + (X_L - X_C)^2} \)

\[ = \sqrt{(5)^2 + (5.87)^2} \]

\[ = 7.71 \, \Omega \]
(a) Current \( I = \frac{V}{Z} = \frac{300}{7.71} = 38.91 \text{ A} \)

(b) Phase angle \( \phi = \tan^{-1} \left( \frac{X_L - X_C}{R} \right) = \tan^{-1} \left( \frac{5.87}{5} \right) = 49.58^\circ \)

(c) Impedance of coil, \( Z_{\text{COIL}} \)
\[
= \sqrt{R^2 + X_L^2} = \sqrt{[(5)^2 + (37.70)^2]} = 38.03 \text{ } \Omega
\]
Voltage across coil \( V_{\text{COIL}} = IZ_{\text{COIL}} \)
\[
= (38.91)(38.03) = 1480 \text{ V}
\]
Phase angle of coil \( = \tan^{-1} \left( \frac{X_L}{R} \right) = \tan^{-1} \left( \frac{37.70}{5} \right) = 82.45^\circ \) lagging
(d) Voltage across capacitor

\[ V_C = I X_C = (38.91)(31.83) \]
\[ = 1239 \text{ V} \]
Problem 16. The following three impedances are connected in series across a 40 V, 20 kHz supply: (i) a resistance of 8 Ω, (ii) a coil of inductance 130 μH and 5 Ω resistance, and (iii) a 10 Ω resistor in series with a 0.25 μF capacitor. Calculate (a) the circuit current, (b) the circuit phase angle and (c) the voltage drop across each impedance.
Inductive reactance, \( X_L = 2\pi fL \)

\[ = 2\pi (20 \times 10^3)(130 \times 10^{-6}) \]
\[ = 16.34 \Omega \]

Capacitive reactance,

\[ X_C = \frac{1}{2\pi fC} \]

\[ = \frac{1}{2\pi (20 \times 10^3)(0.25 \times 10^{-6})} \]
\[ = 31.83 \Omega \]

Since \( X_C > X_L \), the circuit is capacitive (see phasor)

\[ X_C - X_L = 31.83 - 16.34 = 15.49 \Omega. \]
(a) Circuit impedance, \( Z = \sqrt{R^2 + (X_C - X_L)^2} \)
\[= \sqrt{23^2 + 15.49^2} \]
\[= 27.73 \, \Omega \]

Circuit current, \( I = \frac{V}{Z} = \frac{40}{27.73} = 1.442 \, \text{A} \)

(b) From Figure 15.12(c), circuit phase angle
\[\phi = \tan^{-1} \left( \frac{X_C - X_L}{R} \right) \]
i.e. \( \phi = \tan^{-1} \left( \frac{15.49}{23} \right) = 33.96^\circ \) leading

(c) From Figure 15.16(a), \( V_1 = IR_1 = (1.442)(8) \)
\[= 11.54 \, \text{V} \]
\[ V_2 = IZ_2 = I \sqrt{(5^2 + 16.34^2)} = (1.442)(17.09) = 24.64 \text{ V} \]

\[ V_3 = IZ_3 = I \sqrt{(10^2 + 31.83^2)} = (1.442)(33.36) = 48.11 \text{ V} \]

The 40 V supply voltage is the phasor sum of \( V_1 \), \( V_2 \) and \( V_3 \)
7 Series resonance

As stated in Section 7.6, for an $R-L-C$ series circuit, when $X_L = X_C$ (Figure 7.12(d)), the applied voltage $V$ and the current $I$ are in phase. This effect is called series resonance. At resonance:

(i) $V_L = V_C$

(ii) $Z = R$ (i.e. the minimum circuit impedance possible in an $L-C-R$ circuit)
(iii) \[ I = \frac{V}{R} \] (i.e. the maximum current possible in an \[ L-C-R \] circuit)

(iv) Since \( X_L = X_C \), then \[ 2\pi f_r L = \frac{1}{2\pi f_r C} \]

from which, \[ f_r^2 = \frac{1}{(2\pi)^2 LC} \]

and, \[ f_r = \frac{1}{2\pi \sqrt{LC}} \text{ Hz} \]

where \( f_r \) is the resonant frequency.
Problem 18. A coil having a resistance of 10 Ω and an inductance of 125 mH is connected in series with a 60 μF capacitor across a 120 V supply. At what frequency does resonance occur? Find the current flowing at the resonant frequency.

Resonant frequency, \( f_r = \frac{1}{2\pi \sqrt{(LC)}} \) Hz

\[
= \frac{1}{2\pi \sqrt{\left[\left(\frac{125}{10^3}\right)\left(\frac{60}{10^6}\right)\right]}} \text{ Hz}
\]

\[
= \frac{1}{2\pi \sqrt{\left[\left(125\right)\left(6\right)\right]}} = 58.12 \text{ Hz}
\]

At resonance, \( X_L = X_C \) and impedance \( Z = R \)

Hence current, \( I = \frac{V}{R} = \frac{120}{10} = 12 \text{ A} \)
Problem 19. The current at resonance in a series $L-C-R$ circuit is $100 \, \mu A$. If the applied voltage is $2 \, \text{mV}$ at a frequency of $200 \, \text{kHz}$, and the circuit inductance is $50 \, \mu \text{H}$, find (a) the circuit resistance, and (b) the circuit capacitance.

(a) \[ I = 100 \, \mu A = 100 \times 10^{-6} \, \text{A}; \]
\[ V = 2 \, \text{mV} = 2 \times 10^{-3} \, \text{V} \]

At resonance, impedance $Z = \text{resistance } R$

Hence \[ R = \frac{V}{I} = \frac{20 \times 10^{-3}}{100 \times 10^{-6}} = \frac{2 \times 10^{6}}{100 \times 10^{3}} = 20 \, \Omega \]

(b) At resonance $X_L = X_C$

\[ \text{i.e. } 2\pi fL = \frac{1}{2\pi fC} \]
Hence capacitance

\[ C = \frac{1}{(2\pi f)^2 L} \]

\[ = \frac{1}{(2\pi \times 200 \times 10^3)^2 (50 \times 10^{-6})} \text{ F} \]

\[ = \frac{(10^6)(10^6)}{(4\pi)^2 (10^{10})(50)} \mu\text{F} \]

\[ = 0.0127 \mu\text{F} \text{ or } 12.7 \text{ nF} \]
8 Q-factor

Voltage magnification at resonance

\[
\text{voltage across } L \text{ (or } C) = \frac{V_L}{V} = \frac{I X_L}{IR} = \frac{X_L}{R} = \frac{2\pi f_r L}{R}
\]

Hence Q-factor

\[
\frac{V_C}{V} = \frac{I X_C}{IR} = \frac{X_C}{R} = \frac{1}{2\pi f_r CR}
\]

Alternatively, Q-factor

At resonance

\[
f_r = \frac{1}{2\pi \sqrt{(LC)}} \quad \text{i.e.} \quad 2\pi f_r = \frac{1}{\sqrt{(LC)}}
\]

Hence Q-factor

\[
\frac{2\pi f_r L}{R} = \frac{1}{\sqrt{(LC)}} \left( \frac{L}{R} \right) = \frac{1}{R} \sqrt{\left( \frac{L}{C} \right)}
\]
Problem 20. A coil of inductance 80 mH and negligible resistance is connected in series with a capacitance of 0.25 μF and a resistor of resistance 12.5 Ω across a 100 V, variable frequency supply. Determine (a) the resonant frequency, and (b) the current at resonance. How many times greater than the supply voltage is the voltage across the reactances at resonance?

a) Resonant frequency $f_r$

$$f_r = \frac{1}{2\pi \sqrt{\left[\left(\frac{80}{10^3}\right)\left(\frac{0.25}{10^6}\right)\right]}} = \frac{1}{2\pi \sqrt{\left[\frac{(8)(0.25)}{10^8}\right]}}$$

$$= \frac{10^4}{2\pi \sqrt{2}}$$

$$= 1125.4 \text{ Hz} = 1.1254 \text{ kHz}$$
(b) Current at resonance \( I = \frac{V}{R} = \frac{100}{12.5} = 8 \text{ A} \)

Voltage across inductance, at resonance,

\[
V_L = IX_L = (I)(2\pi fL) \\
= (8)(2\pi)(1125.4)(80 \times 10^{-3}) \\
= 4525.5 \text{ V}
\]

(Also, voltage across capacitor,

\[
V_C = IX_C = \frac{I}{2\pi fC} = \frac{8}{2\pi (1125.4)(0.25 \times 10^{-6})} \\
= 4525.5 \text{ V}
\]
Problem 21. A series circuit comprises a coil of resistance $2 \, \Omega$ and inductance $60 \, \text{mH}$, and a $30 \, \mu\text{F}$ capacitor. Determine the Q-factor of the circuit at resonance.

At resonance, Q-factor $= \frac{1}{R} \sqrt{\left( \frac{L}{C} \right)}$

$= \frac{1}{2} \sqrt{\left( \frac{60 \times 10^{-3}}{30 \times 10^{-6}} \right)}$

$= \frac{1}{2} \sqrt{\left( \frac{60 \times 10^6}{30 \times 10^3} \right)}$

$= \frac{1}{2} \sqrt{(2000)} = 22.36$
0.9 Bandwidth and selectivity

\[ Q = \frac{f_r}{f_2 - f_1} \quad \text{or} \quad (f_2 - f_1) = \frac{f_r}{Q} \]
Problem 23. A filter in the form of a series \(L-R-C\) circuit is designed to operate at a resonant frequency of 5 kHz. Included within the filter is a 20 mH inductance and 10 \(\Omega\) resistance. Determine the bandwidth of the filter.

Q-factor at resonance is given by

\[
Q_r = \frac{\omega_r L}{R} = \frac{(2\pi 5000)(20 \times 10^{-3})}{10} = 62.83
\]

Since \(Q_r = f_r/(f_2 - f_1)\)

bandwidth, \((f_2 - f_1) = \frac{f_r}{Q_r} = \frac{5000}{62.83} = 79.6\ \text{Hz}\)
the instantaneous power, \( p = vi \),

(a) For a purely resistive a.c. circuit, the average power dissipated, \( P \), is given by:

\[
P = VI = I^2R = \frac{V^2}{R} \text{ watts}
\]
(b) For a purely inductive a.c. circuit, the average power is zero. See Figure .23(b).

(c) For a purely capacitive a.c. circuit, the average power is zero. See Figure .23(c).
Figure 15.24 shows current and voltage waveforms for an $R-L$ circuit where the current lags the voltage by
11 Power triangle and power factor

Apparent power, \( S = VI \) voltamperes (VA)

True or active power, \( P = VI \cos \phi \) watts (W)

Reactive power, \( Q = VI \sin \phi \) reactive voltamperes (var)

(a) PHASOR DIAGRAM

(b) POWER TRIANGLE
Power factor = \frac{\text{True power } P}{\text{Apparent power } S}

For sinusoidal voltages and currents,

\text{power factor} = \frac{P}{S} = \frac{VI \cos \phi}{VI}, \text{ i.e.}

\text{p.f.} = \cos \phi = \frac{R}{Z}