Sheet No. (7)

(1) Find the second order approximation to the function:

   \( (i) \ f(x, y) = e^{x-2y} \) near the point \((0,0)\).

   \( (ii) \ f(x, y) = \sin(xy) \) near the point \(\left(1, \frac{\pi}{2}\right)\).

(2) Test for maximum and minimum the functions:

   (i) \( z = x^2 + 3xy - 2y^2 + 5x - y + 1 \).

   (ii) \( z = x^3 + y^3 - 3x - 3y \)

   (iii) \( z = x^3 + y^3 - 3xy \).

\[
f(x, y) \approx P_2(x, y) = f(a, b) + \left[ h f_x(a, b) + k f_y(a, b) \right] + \frac{1}{2!} \left[ h^2 f_{xx}(a, b) + 2hk f_{xy}(a, b) + k^2 f_{yy}(a, b) \right] + \ldots
\]
Theorem

Let \( f_{xx} \) and \( f_{yy} \) of the function \( f(x, y) \) exist and, 
\[ f_x(a, b) = 0 \quad \text{and} \quad f_y(a, b) = 0 \]

Define the discriminate \( D \) by:
\[ D = f_{xx}(a, b)f_{yy}(a, b) - (f_{xy}(a, b))^2 \]

then if:
- \( D > 0 \) and \( f_{xx}(a, b) < 0 \) then \( f(a, b) \) is a local maximum.
- \( D > 0 \) and \( f_{xx}(a, b) > 0 \) then \( f(a, b) \) is a local minimum.
- \( D < 0 \) then \( f(a, b) \) is a saddle point (not maximum nor minimum).
- \( D = 0 \) then, this test fail and the function must be investigated near the point \( (a, b) \), but any other test is beyond the scope.