LECTURE
NO. (7)
MATH. (2)
Analytical Geometry

ELLIPSE
**Definition**

An ellipse is the locus (مسار) of a point \(P(x,y)\) moving in a plane such that:

\[
\frac{Distance\ from\ P(x,y)\ to\ a\ focus}{Distance\ from\ P(x,y)\ to\ its\ directrix} = e < 1
\]
Standard Forms of Ellipse equations:

(i) X-Ellipse:

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
\]

\[
b^2 = a^2 \left(1 - e^2\right)
\]

\[
x = -\frac{a}{e}
\]

\[
x = \frac{a}{e}
\]
Standard Forms of Ellipse
equations:

(ii) Y- Ellipse:

\[ \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \]
* The length of the major axis as $2a$ and the length of the minor axis as $2b$.

* The center of the ellipse is the midpoint of the major axis.
* The vertices are the end points of the major axis.
* The foci of the ellipse are on the major axis.

* The length of lutus rectum is \( \frac{2b^2}{a} \)
Example:

Find the center, vertices, axes, foci, directrices, and sketch the ellipse

\[ 49x^2 + 25y^2 = 1225 \]

Solution:

\[ \frac{x^2}{25} + \frac{y^2}{49} = 1 \quad \Rightarrow \quad a = 7, \ b = 5, \quad e \approx 0.7 \]

<table>
<thead>
<tr>
<th>Center</th>
<th>Vertices</th>
<th>Foci</th>
<th>Axes</th>
<th>Directrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>(0,7)</td>
<td>(0,4.9)</td>
<td>x=0</td>
<td>y=10</td>
</tr>
<tr>
<td>(0,-7)</td>
<td>(0,-4.9)</td>
<td></td>
<td>y=0</td>
<td>y=-10</td>
</tr>
</tbody>
</table>
Ellipses with Center at $C(x_0, y_0)$

**X-Ellipse**

$$\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} = 1$$

**Y-Ellipse**

$$\frac{(x - x_0)^2}{b^2} + \frac{(y - y_0)^2}{a^2} = 1$$

**General Equation**

$$ax^2 + by^2 + 2gx + 2fy + c = 0$$

$a \neq b$
Example:

Find the center, vertex, axis, focus, directrix for the ellipse

\[ 5x^2 + 9y^2 - 10x - 54y + 41 = 0 \]

Solution:

\[
\begin{align*}
(5x^2 - 10x) + (9y^2 - 54y) + 41 &= 0 \\
5(x^2 - 2x) + 9(y^2 - 6y) + 41 &= 0 \\
5(x - 1)^2 - 5 + 9(y - 3)^2 - 81 + 41 &= 0 \\
5(x - 1)^2 + 9(y - 3)^2 &= 45 \\
\frac{(x - 1)^2}{9} + \frac{(y - 3)^2}{5} &= 1
\end{align*}
\]
\[
\frac{(x - 1)^2}{9} + \frac{(y - 3)^2}{5} = 1
\]

\[a = 3, \quad b = \sqrt{5}, \quad e = \frac{2}{3}\]

<table>
<thead>
<tr>
<th>Center</th>
<th>Vertex</th>
<th>Focus</th>
<th>Axis</th>
<th>Directrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,3)</td>
<td>(4,3)</td>
<td>(3,3)</td>
<td>y=3</td>
<td>x=5.5</td>
</tr>
<tr>
<td>(-2,3)</td>
<td>(-1,3)</td>
<td>(-1,3)</td>
<td>x=1</td>
<td>x= -3.5</td>
</tr>
</tbody>
</table>
Example:

Find the center, vertex, axis, focus, directrix for the ellipse

\[ 9x^2 + 4y^2 + 36x - 8y + 4 = 0 \]

Solution:

\[
(9x^2 + 36x) + (4y^2 - 8y) + 4 = 0
\]

\[
9(x^2 + 4x) + 4(y^2 - 2y) + 4 = 0
\]

\[
9(x + 2)^2 - 36 + 4(y - 1)^2 - 4 + 4 = 0
\]

\[
9(x + 2)^2 + 4(y - 1)^2 = 36
\]

\[
\frac{(x + 2)^2}{4} + \frac{(y - 1)^2}{9} = 1
\]
\[
\frac{(x + 2)^2}{4} + \frac{(y - 1)^2}{9} = 1
\]

\[
a = 3, \ b = 2, \quad e = \frac{\sqrt{5}}{3}
\]

<table>
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<tr>
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<th>Vertex</th>
<th>Focus</th>
<th>Axis</th>
<th>Directrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-2,1)</td>
<td>(-2,4)</td>
<td>(-2,3.24)</td>
<td>y=1</td>
<td>y=5.02</td>
</tr>
<tr>
<td>(-2,-2)</td>
<td>(-2,-1.24)</td>
<td>(-2,-1.24)</td>
<td>x=-2</td>
<td>y=-3.02</td>
</tr>
</tbody>
</table>
To get the equation ellipse, we must know
-- The type
-- The center
-- The value of \(a, b\)

**Note:**

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**Example:**

Write the equation of the ellipse with center at \((2, -1)\), with major axis = 10 and parallel to the x-axis, and with minor axis = 8.

**Solution:**

-- The type  X-ellipse
-- The center  \((2, -1)\)

\[
\frac{(x - 2)^2}{25} + \frac{(y + 1)^2}{16} = 1
\]
Find the equation of ellipse with, vertices (6, 8), (6, -2) and one foci is (6, 5).

**Solution**

-- The type y-ellipse

-- The center (6,3)

-- The value of $a$, $b$ $2a=10$, $a=5$

$ae=2$, $e=0.4$

$b^2=21$

$$\frac{(x - 6)^2}{21} + \frac{(y - 3)^2}{25} = 1$$
Find the equation of ellipse with, one axis = 18 and the ends points of the other axis are (2, 5), (2, -3).

Solution

-- The type x-ellipse

-- The center (2,1)

-- The value of $a, b$ $2a=18$, $a=9$

$2b=8$, $b=4$

$$
\frac{(x - 2)^2}{81} + \frac{(y - 1)^2}{16} = 1
$$
THE END