CHAPTER 2
DC and AC meters

2.1 Introduction to DC meters

For many decades the basic dc meter movement has been used as the readout device in electronic instrument, even in many instruments that measure ac values. The two most common dc meter movement configurations are the d'Arsonval and the taut-band designs. Both are examples of the permanent magnet moving coil (PMMC) galvanometer, and use the same fundamental principle as the dc motor.

2.2 Moving Coil Instrument

Moving coil instrument or generally referred as the d'Arsonval meter or a permanent magnet moving coil (PMMC) meter is mostly used in direct current measurement. The basic construction of this instrument is shown in Figure 2.1. It is a horseshoe shaped permanent magnet with soft iron pole pieces attached to it. Between the pole pieces is a cylindrical shaped soft iron core about which a coil of fine is wound. This wire is wound on a very light metal frame and mounted in such a setting so that it can rotate freely. A pointer attached to the moving coil deflects up scale as the moving coil rotates.

![d’Arsonval meter](image)

Figure 2.1: d’Arsonval meter

Current from a measured circuit passes through the winding of the moving coil. This current causes the moving coil to behave such as an electromagnet with its own north and south poles. This poles interact with the poles of the permanent magnet causing the coil to rotate. The pointer deflects up scale whenever current flows in the proper direction in the coil. For this reason, all dc meter show polarity markings. It should be emphasized that d'Arsonval meter movement is a current responding device. Regardless of the units for which the scale is calibrated, the moving coil responds to the amount of current through its windings.
The PMMC instrument is essentially a low level dc ammeter; however, with the use of parallel-connected resistor, it can be employed to measure a wide range of direct current levels. The instrument may also be made to function as a dc voltmeter by connecting appropriate value resistor in series with the coil. AC ammeters and voltmeters can be constructed by using rectifier circuits with PMMC instrument. Ohmmeters can be made from precision resistor, PMMC instruments and batteries. Multi-range meters are also available which combine ammeter, voltmeter and ohmmeter functions all in one instrument.

2.1.1 Advantages and disadvantages of moving coil instrument

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2.3 D'Arsonval Meter as a DC Ammeter

Since the windings of the moving coil are constructed of very fine wire, the basic d'Arsonval meter has limited usefulness without modification. One desirable modification is to increase the range of current that can be measured by the basic meter. This is done by placing a low resistance in parallel with moving coil resistance, \( R_m \). This low resistance is called a shunt (\( R_{sh} \)) and its function is to provide an alternative path, for the total meter current \( I \) around the meter movement. The basic dc ammeter circuit is shown in Figure 2.2. In the most circuits, \( I_{sh} \) is much greater than \( I_m \), which flows in the moving coil itself. The resistance of the shunt is found by applying Ohm's Law to Figure 2.2.

![Figure 2.2: Basic dc ammeter circuit](image)
where:

- \( R_{sh} \) = resistance of the shunt
- \( R_m \) = internal resistance of the moving coil
- \( I_{sh} \) = current through the shunt
- \( I_m \) = full scale deflection current of the moving coil
- \( I \) = full scale deflection current of the ammeter
- \( V_m \) = voltage drop across the \( R_m \)
- \( V_{sh} \) = voltage drop across the \( R_{sh} \)

- The voltage drop across the meter movement, \( V_m = I_m R_m \)
- Since the shunt resistor is parallel with the moving coil, \( V_m = V_{sh} \)
- The current through the shunt, \( I_{sh} = I - I_m \)
- Therefore, the value of the shunt is

\[
R_{sh} = \frac{V_{sh}}{I_{sh}} = \frac{\frac{R_m}{I_{sh}}}{I_{sh}} = \frac{R_m}{I_{sh} - I_m}
\]

The purpose of introducing the shunt current is to measure a current \( I \) that is \( n \) times larger than \( I_m \). The number \( n \) is known as multiplying factor and relates total current and moving coil current as

\[
I = nI_m
\]

Substituting this for \( I \) in equation (2.1) yields

\[
R_{sh} = \frac{\frac{R_m}{I_m - I_m}}{I_{sh} - I_m}
\]

\[
R_{sh} = \frac{R_m}{-1} \quad \text{.................................}(2.2)
\]
Example 2.1
A 100µA meter movement with an internal resistance of 800Ω is used in a 0 - 100 mA ammeter. Find the value of the required shunt resistance.

Solution
The multiplication factor $n$ is the ratio of 100 mA to 100µA

$$n = \frac{I}{I_m} = \frac{100 \text{ mA}}{100 \text{ µA}} = 1000$$

Therefore,

$$R_{sh} = \frac{R_m}{n - 1} = \frac{800}{999} = 0.8 \Omega$$

Example 2.2
Calculate the value meter with a 100 Ω of shunt resistance required to convert a 1 mA moving coil meter with a 100 Ω internal resistance into a 0 - 10 mA ammeter using both using both equation (2.1) and (2.2).

Solution

$$V_m = I_m R_m = 1 \text{ mA} \times 100\Omega = 0.1 \text{ V}$$

$$V_{sh} = V_m = 0.1 \text{ V}$$

$$I_{sh} = I - I_m$$

$$= (10 - 1) \text{ mA}$$

$$= 9 \text{ mA}$$

Using equation (2.1)

$$R_{sh} = \frac{V_{sh}}{I_{sh}}$$

$$= \frac{0.1 \text{ V}}{9 \text{ mA}}$$

$$= 11.11 \Omega$$

Using equation (2.2)

$$n = \frac{I}{I_m} = \frac{10 \text{ mA}}{1 \text{ mA}} = 10$$

$$R_{sh} = \frac{(R_m)}{(n - 1)} = \frac{100 \Omega}{(10 - 1)} = 11.11 \Omega$$
2.3.1 Ayrton Shunt

The shunt resistance discussed in the previous sections work well enough on a single range ammeter. However, on a multiple-range ammeter, the Ayrton shunt is frequently a more suitable design. One advantage of the Ayrton shunt is it eliminates the possibility of the moving coil to be in the circuit without any shunt resistance where they protect the deflection instrument of the ammeter from an excessive current flow when switching between shunts. Another advantage is that it may be used as a wide range ammeter. The Ayrton shunt circuit is shown in Figure 2.3.

![Ayrton Shunt Circuit Diagram](image)

Figure 2.3: An ammeter using an Ayrton shunt

The individual resistance values of the shunts are calculated starting from the most sensitive range, $I_1$ and works towards the least sensitive range, $I_3$. At the most sensitive range (point A), the shunt can be computed using equation (2.2), where,

$$R_{sh} = \frac{R_m}{n-1} \hspace{2cm} \text{(2.2)}$$

The equation needed to compute the value of each $R_a$, $R_b$ and $R_c$ can be developed by observing Figure 2.3. Notice that $R_a + R_b + R_c = R_{sh}$

Now, observe at point B,

$$(R_b + R_c) \parallel (R_a + R_m)$$

Then the voltage across the branch can be written as

$$V_{(R_b + R_c)} = V_{(R_a + R_m)}$$
In current and resistance terms we can write as

\[(R_b + R_c)(I_2 - I_m) = I_m (R_a + R_m)\]
\[I_2 (R_a + R_c) - I_m (R_a + R_c) = I_m (R_a + R_m)\]
\[I_2 (R_b + R_c) = I_m (R_a + R_c + I_m R_a + R_m)\]
\[R_b + R_c = I_m (R_{sh} + R_m) / I_2 \] \hspace{1cm} (2.3)

At point C,
\[R_c \parallel (R_a + R_b + R_m)\]

Thus, in current and resistance terms we can write it as

\[V_{Rc} = V_{(R_a + R_b + R_m)}\]
\[(R_c)(I_3 - I_m) = (R_a + R_b + R_m)(I_m)\]
\[I_3 R_c = I_m (R_{sh} + R_m)\]
\[R_c = \frac{I_m (R_{sh} + R_m)}{I_3} \] \hspace{1cm} (2.4)

The value of \(R_b\) can be obtained by substituting equation 2.4 into equation 2.3 which yields to:

\[R_b = I_m (R_{sh} + R_m) \left[ \frac{1}{I_2} - \frac{1}{I_3} \right] \] \hspace{1cm} (2.5)

**Example 2.3**

Calculate the value for \(R_a\), \(R_b\) and \(R_c\) as shown in Figure 2.3, given the value of internal resistance, \(R_m = 1 \text{k} \Omega\) and full scale current of the moving coil = 100 µA. The required range of current are: \(I_1 = 10 \text{ mA}\), \(I_2 = 100 \text{ mA}\) and \(I_3 = 1\text{ A}\).

**Solution**

At the most sensitive range,
\[n = \frac{I}{I_m} = \frac{10 \text{ mA}}{100 \text{ µA}} = 100\]

Total shunt resistance, \(R_{sh} = \frac{R_m}{(n-1)} = \frac{1 \text{k} \Omega}{99} = 10.1 \text{ Ω}\)

Using equation 2.4 (at point C),
\[R_c = (100 \text{ µA}) (10.1 \text{ Ω} + 1 \text{k} \Omega) / 1\text{ A} = 0.101 \text{ Ω}\]
Using equation 2.5,

\[ R_b = (100 \, \mu A)(10.1 \, \Omega + 1k\Omega)[(1/100mA) - (1/1A)] \]
\[ = 0.909 \, \Omega \]

For \( R_a \), since \( R_{sh} = R_a + R_b + R_c \), thus \( R_a = R_{sh} - (R_b + R_c) \)
\[ R_a = 10.1 \, \Omega - (0.909 \, \Omega + 0.101 \, \Omega) \]
\[ = 9.09 \, \Omega \]

### 2.3.2 Ammeter Insertion Effects

All ammeters contain some external resistance, which may range from a low to a greater value. Inserting ammeter in a circuit always increases the resistance of the circuit and therefore reduces the current in the circuit.

Without ammeter, the current in the circuit shown in Figure 2.4 can be calculated as

\[ I_e = \frac{E}{R_1} \]

\[ \text{……………………………… (2.6)} \]

![Figure 2.4: Circuit without ammeter insertion effects](image)

Inserting the ammeter as in Figure 2.5 (X-Y terminal), will reduce the current in the circuit to:

\[ I_m = \frac{E}{R_1 + R_m} \]

\[ \text{……………………………… (2.7)} \]
Dividing equation (2.7) by equation (2.6) gives the following expression:

\[
\frac{I_m}{I_e} = \frac{R_1}{R_1 + R_m} 
\]

......................... (2.8)

Insertion error = \(\frac{I_e - I_m}{I_e} \times 100\%\) 

......................... (2.9)

**Example 2.4**

Determine the insertion error due to using an ammeter in Figure 2.5 if \(E = 100\, V\), \(R_1 = 100\, \Omega\) and \(R_m = 100\, \Omega\).

**Solution**

\(I_e = \frac{E}{R_1} = \frac{100\, V}{100\, \Omega} = 1\, A\)

\(I_m = \frac{E}{(R_1+R_m)} = \frac{100\, V}{(100\, \Omega+100\, \Omega)} = 0.5\, A\)

Insertion error = \((1-0.5)/1 \times 100\% = 50\%\)

**2.4 D’Arsonval Meter as a DC Voltmeter**

The basic d’Arsonval meter can be converted to a dc voltmeter by connecting a multiplier \(R_s\) in series with it as shown in Figure 2.6. The purpose of the multiplier is to extend the range of the meter and to limit the current through the d’Arsonval meter to the maximum full-scale deflection current.

To find the value of the multiplier resistor, we may first determine the sensitivity, \(S\), of the d’Arsonval. If the sensitivity is known, the total voltmeter resistance can be calculated easily. The sensitivity of a voltmeter is always specified by the manufacturer, and is frequently printed on the scale of the instrument. If the full-scale meter current is known, the sensitivity can be determined as the reciprocal of the full scale current.

\[\text{Sensitivity} = \frac{1}{I_{fs}}\]  

......................... (2.10)

Where \(I_{fs}\) is the full-scale deflection current of d’Arsonval meter.
The value of the multiplier resistance can be found using this relationship:

\[ R_s + R_m = S \times V_{\text{range}} \]  

……………………………… (2.11)

Thus, \[ R_s = S \times V_{\text{range}} - R_m \]  

……………………………… (2.12)

**Example 2.5**

Calculate the value of the multiplier resistance on the 50 V range of a dc voltmeter that used a 500µA d’Arsonval meter with an internal resistance of 1 kΩ.

**Solution**

\[ S = \frac{1}{I_{fs}} = \frac{1}{500 \, \mu A} = 2 \, k\Omega/V \]

\[ R_s = S \times \text{Range} - R_m \]
\[ = 2 \, k\Omega/V \times 50 \, V - 1 \, k\Omega \]
\[ = 99 \, k\Omega \]

**2.4.1 Multi-Range Voltmeter**

A multirange voltmeter consists of a deflection instrument, several multiplier resistors and a rotary switch. Two possible circuits are illustrated in Figure 2.7,
In figure 2.7(a) only one of the three multiplier resistors is connected in series with the meter at any time. The range of this meter is

\[ V = \text{Im}(Rm + R) \]

Where the multiplier resistance, \( R \) can be \( R_1 \) or \( R_2 \) or \( R_3 \)

In figure 2.7(b) the multiplier resistors are connected in series, and each junction is connected to one of the switch terminals. The range of this voltmeter can be also calculated from the equation

\[ V = \text{Im}(Rm + R) \]

Where the multiplier, \( R \), now can be \( R_1 \) or \( (R_1 + R_2) \) or \( (R_1 + R_2 + R_3) \)

(Note: the largest voltage range must be associated with the largest sum of the multiplier resistance)

Of the two circuits, the one in Figure 2.7(b) is the least expensive to construct. All the multiplier resistor in Figure 2.7(a) must be special (non-standard) values, whereas in Figure 2.7(b) only \( R_1 \) is a special resistor and all other multiplier are standard-value (precise) resistor.

**Example 2.6**

Calculate the value of the multiplier resistance for the multiple range dc voltmeter circuit shown in Figure 2.7(a) and Figure 2.7(b), if \( I_s = 50\mu A \) and \( R_m = 1k\Omega \)
2.4.2 DC Voltmeter Loading Effect

As the DC ammeter, the DC voltmeter also observe for loading effect whenever it is inserted to a measured circuit. Figure 2.7 shows a circuit with the DC voltmeter is inserted into it. Inserting voltmeter always increase the resistance and decrease the current flowing through the circuit.

Without the insertion of the DC voltmeter, the voltage \( V_{RB} \) can be found as:

\[
V_{RB} = \frac{R_B}{R_A + R_B} x E
\]

……………………… (2.13)

Inserting the voltmeter in parallel with \( R_B \) gives us the total inserted resistance as:

\[
R_T = R_s + R_m
\]

……………………… (2.14)

Thus, yield to

\[
R_{eq} = R_B \parallel R_T
\]

……………………… (2.15)

Now, the voltage \( V_{RB}^m \) with the voltmeter insertion is found as:

\[
V_{RB}^m = \frac{R_{eq}}{R_{eq} + R_A} x E
\]

……………………… (2.16)

Therefore,

\[
\text{Insertion error} = \frac{V_{RB} - V_{RB}^m}{V_{RB}} \times 100\%
\]

……………………… (2.17)
2.5 D’Arsonval Meter as a DC Ohmmeter

The d’Arsonval meter can also be transformed as an ohmmeter for any circuit’s resistance measurement. The basic ohmmeter circuit employing d’Arsonval meter is shown in Figure 2.8:

Before $R_x$ is measured, the ohmmeter must first be calibrated. The “zero” calibration is performed by shorting the terminal x-y and adjusting $R_z$ so that full-scale deflection on the meter movement is obtained. Without $R_x$, the current equation for full-scale deflection becomes

$$I_{fs} = \frac{E}{R_z + R_m} \quad \ldots \quad (2.18)$$

With $R_x$, the current equation will now becomes:

$$I = \frac{E}{R_x + R_z + R_m} \quad \ldots \quad (2.19)$$

Notice that, by introducing $R_x$, the current $I$ will always less than the full-scale current

$$I < I_{fs}$$

The relationship between the full-scale deflection with the value of $R_x$ can be given as

$$P = \frac{I}{I_{fs}} = \frac{R_z + R_m}{R_x + R_z + R_m} \quad \ldots \quad (2.20)$$

Practically, this equation is being used to develop the marking scale on the meter face to indicate the value of the measured resistor.
2.5.1 Multiple-range Ohmmeters

The ohmmeter range discussed in the previous section is not capable of measuring resistance over wide range of values. Therefore, we need to extend our discussion of ohmmeters to include multiple-range ohmmeters. Figure 2.9 shows one way of circuit configuration for developing multi-range ohmmeter.

To simplify our analysis, let’s the value each components be

\[
\begin{align*}
R_1 &= 10\Omega \\
R_2 &= 100\Omega \\
R_3 &= 1k\Omega \\
R_m &= 2k\Omega \\
R_z &= 28k\Omega \\
I_{fs} &= 50uA \\
E &= 1.5V
\end{align*}
\]

This instrument makes use of a basic 50uA meter movement with an internal resistance of 2kΩ. An additional resistance of 28kΩ is provided by Rz. Rz is necessary to limit current through the meter movement to 50uA when the test probes (not shown) connected to X and Y are shorted. As may be seen, when the instrument is on the Rx1 range, a 10Ω resistor is in parallel with the meter movement. Therefore, the internal resistance of the ohmmeter on the Rx1 range is 10Ω||30kΩ which approximately 10Ω. This means the pointer will deflect to mid-scale when a 10Ω resistor is connected across X and Y.

When the instrument is set to Rx10 range, the total resistance of the ohmmeter is 100Ω||30kΩ, which is now approximately 100Ω. Therefore, the pointer deflects to mid-scale when 100Ω resistor is connected between the test probes. Mid-scale is marked as 10Ω. Therefore, the value of the resistor is determined by multiplying the reading by the range multiplier of 10 producing a midscale value of 100Ω (Rx100)

When our ohmmeter is set on the Rx100 range, the total resistance of the instrument is 1kΩ||30kΩ which is still approximately 1kΩ. Therefore the pointer deflects to mid-scale when we connect the test probes across a 1-kΩ resistor. This provides us a value for the mid-scale reading of 10 multiplied by 100, or 1kΩ for our resistor.
Example 2.6

(a) In figure below, determine the current through the meter, \( I_m \), when a 20\( \Omega \) resistor between the terminal X and Y are measured on the Rx1 range.

(b) Show that this same current flows through the meter movement when a 200\( \Omega \) resistor is measured on the Rx10 range.

(c) Show that the same current flows when a 2k\( \Omega \) is measured on the Rx100 range.

**Figure 2.10:** Circuit for Example 2.6 with ohmmeter on Rx1 range.

**Solution:**

(a) When the ohmmeter is set on the Rx1 range, the circuit is as shown above. The voltage across the potential combination of resistance computed as

\[
V = \frac{10\Omega}{10\Omega + 20\Omega} \times 1.5V = 0.5V
\]

The current through the meter movement is computed as

\[
I_m = \frac{0.5V}{30k\Omega} = 16.6\mu A
\]

(b) When the ohmmeter is set on the Rx10 range, the circuit is as shown in Figure 2.11. The voltage across the parallel combination of resistance computed as

\[
V = \frac{100\Omega}{100\Omega + 200\Omega} \times 1.5V = 0.5V
\]

The current through the meter movement is computed as

\[
I_m = \frac{0.5V}{30k\Omega} = 16.6\mu A
\]

**Figure 2.11:** Ohmmeter on Rx10 range

(c) When the ohmmeter is set on the Rx100 range, the circuit is as shown in Figure 2.12. The voltage across the parallel combination is computed as

\[
V = \frac{1k\Omega}{1k\Omega + 2k\Omega} \times 1.5V = 0.5V
\]
The current through the meter movement is computed as

\[ I_m = \frac{0.5V}{30k\Omega} = 16.6\mu A \]

**Figure 2.12:** Ohmmeter on Rx100 range.