DC Chopper

- Prof. Dr. Fahmy El-khouly
Definitions:

The power electronic circuit which converts directly from dc to dc is called *dc-to-dc converter or dc-chopper*.

Chopper is a dc to dc transformer: The input dc voltage can be increased (step-up) or decreased (step-down) in output side so a dc chopper circuit can be considered as dc equivalent to an *transformer*.

Using Semiconductor Devices in Chopper:
(1) Power BJT,
(2) Power MOSFET,
(3) GTO, or
(4) Forced-commutated thyristor.
Principle of Step-Down Operation with Resistive Load

When switch SW is closed for a time $t_1 (or \ t_{on})$, the input voltage $V_s$ appears across the load. If the switch remains off for a time $t_2 (or \ t_{off})$, the voltage across the load is zero.
The average output voltage is given by:

practical devices have a finite voltage drop ranging from 0.5 to 2 V, and for the sake of simplicity we shall neglect the voltage drops of these power semiconductor devices.
The average output current is given by:

\[ V_a = \frac{1}{T} \int_0^{t_1} v_0 \, dt = \frac{t_1}{T} V_s = f t_1 V_s = k V_s \]

\[ I_a = \frac{V_a}{R} = k \frac{V_s}{R} \]

Where, \( T \) is the chopping period, \( t_1 \) (or \( t_{on} \)) is on-time, \( t_2 \) (or \( t_{off} \)) is off-time, \( k = \frac{t_1}{T} \) is duty-cycle, and \( f = \frac{1}{T} \) is the chopping frequency. Thus, \( t_1 \) (or \( t_{on} \)) = \( kT \); and \( t_2 \) (or \( t_{off} \)) = \( (1 - k)T \);
The rms value of output voltage is found from

\[ V_o = \left( \frac{1}{T} \int_0^{kT} v_0^2 \, dt \right)^{1/2} = \sqrt{k} \, V_s \]

Assuming a lossless converter, the input power to the converter is the same as the output power and is given by

\[ P_i = \frac{1}{T} \int_0^{kT} v_0 i \, dt = \frac{1}{T} \int_0^{kT} \frac{v_0^2}{R} \, dt = k \frac{V_s^2}{R} \]

The effective input resistance seen by the source is

\[ R_i = \frac{V_s}{I_s} = \frac{V_s}{kV_s/R} = \frac{R}{k} \]

which indicates that the converter makes the input resistance \( R_i \) as a variable resistance of \( R/k \).
Let, \( v_{ch} \) is the voltage drop across the switch when the switch remains on.

The average output voltage is given by:

\[
V_a = \frac{1}{T} \int_{0}^{t_1} v_{odt} = \frac{t_1}{T} (V_s - v_{ch}) = f t_1 (V_s - v_{ch}) = k (V_s - v_{ch})
\]

The average output current is given by:

\[
I_a = \frac{V_a}{R} = \frac{k (V_s - v_{ch})}{R}
\]

The rms value of output voltage is found from

\[
V_o = \sqrt{\frac{1}{T} \int_{0}^{t_1} v_{odt}^2} = \sqrt{\frac{t_1}{T} (V_s - v_{ch})} = \sqrt{f t_1 (V_s - v_{ch})} = \sqrt{k (V_s - v_{ch})}
\]
The output power is given by:

\[ P_o = \frac{1}{T} \int_{0}^{kT} v_o \, idt = \frac{1}{T} \int_{0}^{kT} \frac{v_o^2}{R} \, idt = k \frac{(V_s - v_{ch})^2}{R} \]

The input power is given by:

\[ P_i = \frac{1}{T} \int_{0}^{kT} V_s \, idt = \frac{1}{T} V_s \int_{0}^{kT} \frac{v_o}{R} \, dt = k \frac{V_s (V_s - v_{ch})}{R} \]

If the chopper is lossless then \( P_i \) is equal to \( P_o \) [i.e. \( P_i = P_o \)] and \( v_{ch} = 0 \).

The efficiency is given by:

\[ \eta = \frac{P_o}{P_i} = \left[ 1 - \frac{v_{ch}}{V_s} \right] \]
The effective input resistance is given by:

\[ R_i = \frac{(V_s - v_{ch})}{I_a} = \frac{(V_s - v_{ch})}{k(V_s - v_{ch})/R} = R/k \]

**Example 5.1  Finding the Performances of a Dc–Dc Converter**

The dc converter in Figure 5.1a has a resistive load of \( R = 10 \, \Omega \) and the input voltage is \( V_s = 220 \, V \). When the converter switch remains on, its voltage drop is \( v_{ch} = 2 \, V \) and the chopping frequency is \( f = 1 \, kHz \). If the duty cycle is 50\%, determine (a) the average output voltage \( V_o \), (b) the rms output voltage \( V_{oRMS} \), (c) the converter efficiency, (d) the effective input resistance \( R_i \) of the converter.
Solution

$V_s = 220 \text{ V}, \; k = 0.5, \; R = 10 \; \Omega, \; \text{and} \; v_{ch} = 2 \; \text{V}.$

a. From Eq. (5.1), $V_a = 0.5 \times (220 - 2) = 109 \; \text{V}.$

b. From Eq. (5.2), $V_o = \sqrt{0.5} \times (220 - 2) = 154.15 \; \text{V}.$

c. The output power can be found from

$$P_o = \frac{1}{T} \int_0^T \frac{v_o^2}{R} \, dt = \frac{1}{T} \int_0^T \frac{(V_s - v_{ch})^2}{R} \, dt = k \frac{(V_s - v_{ch})^2}{R}$$

$$= 0.5 \times \frac{(220 - 2)^2}{10} = 2376.2 \; \text{W}$$

The input power to the converter can be found from

$$P_i = \frac{1}{T} \int_0^T V_s i \, dt = \frac{1}{T} \int_0^T \frac{V_s (V_s - v_{ch})}{R} \, dt = k \frac{V_s (V_s - v_{ch})}{R}$$

$$= 0.5 \times 220 \times \frac{220 - 2}{10} = 2398 \; \text{W}$$
The converter efficiency is

\[
\frac{P_o}{P_i} = \frac{2376.2}{2398} = 99.09\%
\]

d. From Eq. (5.4), \( R_i = \frac{10}{0.5} = 20 \, \Omega \).
Control of Duty Cycle of Step-Down Chopper

The duty cycle $k$ can be controlled by the following two ways: (i) Constant-Frequency Operation, and (ii) Variable-Frequency Operation.

Constant-Frequency Operation: The chopping frequency $f$ (or chopping period $T$) is kept constant and the on-time $t_{on}$ is varied. The width of the pulse is varied and this type of control is known as pulse-width-modulation (PWM) control.
Variable-Frequency Operation: The hopping frequency $f$ (or hopping period $T$) is variable and the on-time $t_1$ or off-time $t_2$ is kept constant. This is called frequency modulation.
Disadvantages of frequency modulation control strategy compared to pulse-width modulation control:

(i) Filter design for wide frequency variation is quite difficult.

(ii) There is a possibility of interference with signaling and telephone lines in frequency modulation techniques due to the wide variation of frequency.

(iii) The large off time in frequency modulation technique may make the load current discontinuous, which is undesirable. Thus, the Pulse Width Modulation (constant frequency) system is the preferred scheme for chopper drives.
Principle of Step-UP Operation

When switch SW is closed for a time $t_1$, the inductor current rises and energy is stored in inductor $L$.

If switch is opened for a time $t_2$, the energy stored in the inductor is transferred to load through $D_1$ and the inductor current falls.
When the chopper is turned-on, the voltage across the inductor is:

\[ v_L = L \frac{di}{dt} \]

Assuming that the load current rises linearly from \( I_1 \) to \( I_2 \), and this gives the peak-to-peak ripple current \([\Delta I = I_2 - I_1]\) in the inductor as:

\[ \Delta I = \frac{V_s}{L} t_1 \]

The instantaneous output voltage is:

\[ v_O = V_s + L \frac{\Delta I}{t_2} = V_s \left(1 + \frac{t_1}{t_2}\right) = V_s \frac{1}{1-k} \]

If a large capacitor \( C_L \) is connected across the load as shown by dashed lines in Fig. 9-4(a), the output voltage will be continuous and \( V_o \) would become the average value \( V_a \)
It is seen from above equation that the voltage across the load can be stepped up varying the duty cycle $k$, and the minimum output voltage is $V_s$ when $k = 0$. However, the chopper cannot be switched on continuously such that $k = 1$.

For values of $k$ tending to unity, the output voltage becomes very large and is very sensitive to changes in $k$, as shown in Fig. 9-4(c).
Energy Transfer Between Two Sources Using Chopper

The step-up principle can be applied to transfer energy from one voltage source to another as shown in Fig. 9-5(a).

The equivalent circuits for the mode of operation are shown in Fig. 9-5(b) and the current waveforms in Fig. 9-5(c).
The inductor voltage for mode 1 is given by:

\[ V_S = L \frac{di_1}{dt} \]

The current expression is given as:

\[ i_1(t) = \frac{V_S}{L} t + I_1 \]

Where \( I_1 \) is the initial current for mode 1.

During mode 1, the current must rise and the necessary condition:

\[ \frac{di_1}{dt} > 0 \quad \text{or} \quad V_S > 0 \]

The current for mode 2 is given by:

\[ V_S = L \frac{di_2}{dt} + E \]
The solution of the above equation is:  
\[ i_2(t) = \frac{V_s - E}{L} t + I_2 \]
Where \( I_2 \) is the initial current for mode 2.

For the stable system, the current must fall and the condition is:
\[ \frac{di_2}{dt} < 0 \quad \text{or} \quad V_s < E \]

If this condition is not satisfied, the inductor current would continue to rise and an unstable situation would occur.
The conditions for controllable power transfer are: 
\[ 0 < V_s < E \]

Above Equation indicates that the source voltage \( V_s \) must be less than the voltage \( E \) to permit transfer of power from a fixed (or variable) source to a fixed dc voltage. When the chopper is turned on, the energy is transferred from the voltage source \( V_s \) to inductor \( L \). If the chopper is then turned off, a magnitude of the energy stored in the inductor is forced to battery \( E \). Without the chopping action, \( V_s \) must be greater than \( E \) for transfer power from \( V_s \) to \( E \).
Circuit Operation of a Buck Regulator

The circuit operation can be divided into two modes. Mode 1 begins when $Q_1$ is switched on at $t = 0$. The input current, which rises, flows through the filter inductor $L$, filter capacitor $C$, and load resistor $R$. 

Mode 1
Mode 2 begins when $Q_1$ is switched off at $t = t_2$. The freewheeling diode $D_m$ conducts due to energy stored in the inductor and the inductor current continues to flow through the filter inductor $L$, filter capacitor $C$, load resistor $R$ and diode $D_m$. The inductor current falls until $Q_1$ is switched on again in the next cycle.
The waveforms for the voltages and currents are shown in Fig. 9-12(c) for a continuous current flow in the inductor $L$.

Depending on the switching frequency, filter inductance, filter capacitance, the inductor current could be discontinuous.
The voltage across the inductor $L$ is:

Assuming that the inductor current rises linearly from $I_1$ to $I_2$, in time $t_1$:

$$t_1 = L \frac{I_2 - I_1}{V_s - V_a} = \frac{L \Delta I}{V_s - V_a} \quad (9.31)$$

Where $\Delta I = I_2 - I_1$ is the peak-to-peak ripple current.

Assuming that the inductor current falls linearly from $I_2$ to $I_1$, in time $t_2$:

$$-V_a = L \frac{I_1 - I_2}{t_2} = -L \frac{\Delta I}{t_2}$$

$$t_2 = \frac{L \Delta I}{V_a}$$

substituting

$$t_1 = kT$$

$$t_2 = (1-k)T$$

$$V_a = \frac{t_1}{T} V_s = kV_s$$

Assuming a lossless circuit, $V_s I_s = V_a I_a$  

$= kV_s I_a$ and the average input current:
The switching period $T$ can be expressed as

$$T = t_1 + t_2 = \frac{L\Delta I}{V_s - V_a} + \frac{L\Delta I}{V_a} = \frac{LV_s \Delta I}{(V_s - V_a)V_a}$$

The peak-to-peak ripple current of inductor is

$$\Delta I = \frac{(V_s - V_a)V_a}{fL V_s} = \frac{V_s k (1 - k)}{fL}$$

Using KCL, we can write the inductor current $i_L$ as:

$$i_L = i_c + i_o$$
The buck regulator requires only one BJT, is simple, and has high efficiency greater than 90%.

The $di/dt$ of the load current is limited by inductor $L$. However, the input current is discontinuous and a smoothing input filter is normally required.

It provides one polarity of output voltage and unidirectional output current.

It requires a protection circuit in case of possible short-circuit across the diode path.
In a boost regulator, the average output voltage $V_a$, is greater than the input voltage $V_s$, hence the name “boost,”.

The circuit diagram of a boost regulator using a power MOSFET is shown in Fig. 9-13(a), and this is like a step-up chopper.
The circuit operation can be divided into two modes. Mode 1 begins when $M1$ is switched on at $t = 0$. The input current, which rises, flows through the inductor $L$, and $M1$. 
Mode 2 begins when $M1$ is switched off at $t = t1$. The freewheeling diode $Dm$ conducts due to energy stored in the inductor and the inductor current continues to flow through the filter inductor $L$, filter capacitor $C$, load and diode $Dm$. The inductor current falls until $M1$ is switched on again in the next cycle. The energy stored in the inductor $L$ is transferred to the load.
Mode 2

$V_s$, $i_L$, $L$, $i_1$, $D_m$, $i_c$, $i_o = I_a$

$V_D$

$t_1$, $t_2$, $kT$, $T$

$I_a$, $i_L$, $I_2$, $I_L$, $I_1$

$0$, $kT$, $T$, $t$
Assuming that the inductor current rises linearly from $I_1$ to $I_2$ in time $t_1$

$$t_1 = L \frac{I_2 - I_1}{V_s} = \frac{L \Delta I}{V_s}$$

Assuming that the inductor current falls linearly from $I_2$ to $I_1$ in time $t_2$

$$V_s = L \frac{I_2 - I_1}{t_1} = L \frac{\Delta I}{t_1}$$

Where $\Delta I = I_2 - I_1$ is the peak-to-peak ripple current.

$$V_s - V_a = -L \frac{\Delta I}{t_2} \quad t_2 = \frac{L \Delta I}{V_a - V_s}$$
Equation Eqs. (9.42) and (9.44) and substituting \( t_1 = kT \) and \( t_2 = (1-k)T \), the average output voltage is obtained as follows:

\[
V_a = \frac{T}{t_2} V_s = \frac{V_s}{1-k}
\]

Assuming a lossless circuit, \( V_s I_s = V_a I_a = (V_s I_a)/(1-k) \) and the average input current:

\[
I_s = \frac{I_a}{1-k}
\]

The switching period \( T \) can be expressed as:

\[
T = t_1 + t_2 = \frac{L \Delta I}{V_s} + \frac{L \Delta I}{V_s - V_a} = \frac{LV_a \Delta I}{(V_s - V_a)V_s}
\]
The peak-to-peak ripple current of inductor is:

\[ \Delta I = \frac{(V_a - V_s)V_s}{fL} = \frac{V_s k}{fL} \]
A boost regulator can be step up output voltage without a transformer. Due to single MOSFET, it has high efficiency.

The input current is continuous. However, a high peak current has to flow through the MOSFET. The output voltage is very sensitive to changes in duty cycle $k$ and it might be difficult to stabilize the regulator.

The average output current is less than the average inductor current by a factor of $(1-k)$, and a much higher rms current would flow through the filter capacitor, resulting in the use of a larger filter capacitor and a larger inductor than those of a buck regulator.